# Magnetic Multiscale Model for Local Eddy Current Flow in Complex Materials With Insulated Conductive Particles

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This paper proposes an equivalent magnetic hysteresis (B–H) curve for magnetic multiscale problems of complex materials composed of electrical conductors separated by insulating material. The conventional method, which assigns equivalent material constants to each material, cannot properly express the local eddy current loss and the extra magnetic field derived from the local eddy current. To solve this problem, we derive the magnetic hysteresis curve from the volume-averaged magnetic flux density and the surface magnetic field in the micro-model, and apply this curve to the macro-model. The mesh number of the novel macro-model is reduced to around one-tenth that of the micro-model. Magnetic hysteresis is treated by a free energy magnetic hysteresis model, and the macro-model ignores electrical conductivity. The magnetic hysteresis curve and the numerically calculated losses are consistent in the micro- and macro-models. The proposed method well expresses the local eddy current phenomena.

Index Terms—Homogeneous method, local eddy current, magnetic hysteresis, magnetic multiscale.

## I. INTRODUCTION

AGNETIC multiscale modeling is expected to become an important material design method for complex materials [1]. As modern power electronics enable high-frequency and high-electrical power operation, new magnetic materials are urgently required for small-size transformers and electrical circuits [2].

The design of such complex magnetic materials can be aided by numerical multiscale calculations and homogenization techniques of the magnetic phenomena [3]–[8]. For this purpose, the relationship between the micro- and macro-models must be clarified, as shown in Fig. 1. When the micro-structure in an electromagnetic numerical calculation is expressed in a macro-model, the mesh number and calculation time can become prohibitively large.

In previous work [7], these problems have been resolved by equivalent B-H curves obtained from experimental initial magnetization curves. This approach captures both the nonlinearity and the magnetization anisotropy of the magnetic body in the microstructure [7].

In another study [8], the electromagnetic properties of the micro structure are represented by the equivalent electromagnetic material constants in a micro-model, namely, the equivalent dielectric constant, magnetic permeability, and electrical conductivity (denoted as  $[\hat{e}]$ ,  $[\hat{\mu}]$ ,  $[\hat{\sigma}]$ , respectively). However, [8] reported difficulty in expressing the local eddy current by the equivalent material constants, because this current flows through the microstructure of the complex material, especially in high-frequency operation. This phenomenon will be discussed in detail in Section II-C.

To solve the problem of the local eddy current and the additional magnetic field derived from that current in the micromodel, we focus on equivalent magnetic hysteresis curves.

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Fig. 1. Relationship between micro- and macro-models in magnetic multiscale problems.

 TABLE I

 MATERIAL CONSTANTS FOR MICROMODEL (PRECISE MODEL)

		Unit
Particle size	4×4×4	mm
Particle pitch(Basic structure)	5×5×5	mm
Electrical conductivity: $\sigma$	$3.77 \times 10^{7}$	S/m
Relative magnetic permeability: $\mu_r$	1	-

We then derive the equivalent magnetic hysteresis curve in the macro-model from the volume-averaged magnetic flux density and the surface magnetic field in the micro-model. Using the equivalent magnetic hysteresis curve in the macro-model, we also discuss the iron losses in the numerical calculation. The models are detailed in the following sections.

#### II. MICRO-MODEL

## A. Calculation Model

Fig. 2 shows the micro-model to be solved in this paper. Three cubic electrically conductive particles (each of volume  $4 \text{ mm}^3$ ) are serially arranged at a pitch of 5 mm. The particles are electrically insulated. The unit structure is a 5-mm<sup>3</sup> volume containing an electrically conductive particle.

The basic structures extend to infinity in the x-, y-, and z-directions and can be regarded as a homogeneous material with equivalent material constants. The xy plane is subjected to symmetric boundary conditions and the zx plane has a natural

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Fig. 2. Micro-model at a frequency of 10 kHz (element number: 148 225 and node number: 138 720). (a) Schematic view. (b) Size view and divided mesh.

boundary condition in Fig. 2(a). Therefore, the magnetic field is uniformly supplied to the insulated conductive particles in the y-direction, and then, the basic structures extend to infinity in the y-, and z-directions. Because of computational restraints, we here limit the micro-model to three basic structures in the x-direction. The eddy current flows uniformly in the z-direction at the front end of the x-direction at a frequency of 10 kHz. Details of the material constants are listed in Table I.

By using the  $j\omega$  method, the finite-element analysis is performed, considering the eddy current but not the displacement current. Fig. 2(b) shows the divided mesh shape. As the skin depth is 0.58 mm at 10 kHz, and we are investigating the induced eddy current effect, the fine mesh in the micro-model contains 148 225 elements.

Eddy current flows only within the electrical conductive particles; eddy current between the particles is blocked by the electrical insulation.

## B. Calculation Results

The calculated magnetic flux density and eddy current are shown in Figs. 3 and 4, respectively. Because of the skin depth, the eddy current and magnetic flux density are concentrated at the surface of the particles, and are non-uniformly distributed. Being a primary delay system, the magnetic flux density is large at  $\omega t = \pi/2$ , and the eddy current is large at  $\omega t = 0$ . There is no eddy current between the particles, because they are electrically insulated.

#### C. Discussion on Equivalent Material Constants

Fujisaki and Ikeda [8] determined their equivalent electromagnetic material constants by two methods: 1) a volumeaveraged method and 2) a standing-wave method. The former method has proven useful in multiscale problems of static magnetic fields [1]. The latter, derived from a method that



Fig. 3. Magnetic flux density distribution in three basic structures calculated in the micro-model. (a)  $\omega t = 0$  (real part). (b)  $\omega t = \pi/2$  (imaginary part).



Fig. 4. Eddy current distribution in three basic structures calculated in the micro-model calculation. (a)  $\omega t = 0$  (real part). (b)  $\omega t = \pi/2$  (imaginary part).

measures the high frequency electromagnetic material constants, introduces the equivalent dielectric constant  $\hat{\varepsilon}_{ave}$  and the magnetic permeability  $\hat{\mu}_{ave}$ .

In this previous study, the equivalent material constants were computed by three energy consumption evaluations using the volume-averaged method, the standing-wave method, and the macro-model. Water and aluminum were assumed as the electromagnetic materials. The water and aluminum cases represent the dielectric material condition and the local eddy current condition, respectively.

All three calculations obtained the same energy consumption for the water particles, but diverged in the aluminum particle case. We concluded that the conventional methods for obtaining equivalent material constants (volume-averaged and standing-wave methods) are applicable only to static magnetic fields or dielectric materials, and are unsuitable for local eddy current problems. Therefore, the equivalent electromagnetic constants in local eddy current problems require a new calculation technique.

In the local eddy problem of Fig. 2, the eddy current appears only in the micro-model, and is absent in the macro-model. As the particles are electrically conductive but also electrically insulated from each other, their equivalent electrical conductivities are difficult to assign, and simply become zero in the macro-model.

However, the insulation means that the particle undergoes no Joule's loss and produces no extra magnetic field derived from the eddy current. Therefore, the Joule's loss and the magnetizing property change should be considered by the equivalent material constants in the macro-model.

To solve this problem, we introduce new equivalent electromagnetic material constants as an equivalent magnetic



Fig. 5. Equivalent magnetic hysteresis (B-H) curve of basic structure.

TABLE II MATERIAL CONSTANTS OF MACRO-MODEL (HOMOGENOUS STRUCTURE)

		unit
Electrical conductivity	0	S/m
Relative magnetic permeability	<i>B–H</i> curve of Fig. 5	-

hysteresis curve. Through the magnetic hysteresis curve, we can express local losses, such as local eddy currents, and the changes in the magnetizing property.

The average magnetic flux density is calculated by a conventional approach, namely, by the volume-averaged method as follows:

$$B_{\text{ave}} = \frac{\sum_{\text{Basicstructure}} B_i \Delta V_i}{\sum_{\text{Basicstructure}} \Delta V_i}.$$
 (1)

The average magnetic field is then determined by the surface volume-averaged method. This method is identical to a conventional magnetic field measurement called the *H*-coil method

$$H_{\text{ave}} = \frac{\sum_{\text{SurfaceofBasicstructure}} H_i \Delta V_i}{\sum_{\text{SurfaceofBasicstructure}} \Delta V_i}.$$
 (2)

The calculation results of (1) and (2) are shown in Fig. 5. Magnetic hysteresis curves are generated by local eddy currents in the aluminum particles. The magnetic property varies, because the local eddy current introduces an additional magnetic field. The magnetic hysteresis curve presents as an inclining ellipse, because the volume-averaged magnetic field and volume-averaged magnetic flux density are linearly related.

## III. MACRO-MODEL

#### A. Calculation Model

The macro-model is treated by a homogeneous method, as shown in Fig. 6 [9]. The macro-model assumes the same size, boundary conditions, and exciting current as the micro-model, but the three basic structures containing electrically conductive particles are replaced by a homogenous structure. The electromagnetic property is the magnetic hysteresis (B-H) curve of Fig. 5, and the electrical conductivity is zero (insulator). Table II lists the material constants of the homogenous structure.



(b) Size view and divided mesh.

Fig. 6. Macro-model at a frequency of 10 kHz (element number: 12 825 and node number: 10976). (a) Schematic view. (b) Size view and divided mesh.



Fig. 7. Magnetic flux density distribution in the homogeneous structure of the macro-model. (a)  $\omega t = 0$ . (b)  $\omega t = \pi/2$ .



Fig. 8. Comparison of magnetic hysteresis curves constructed in the microand macro-models.

No eddy current is induced in the homogenous structure, and the skin depth at 10 kHz is irrelevant. Therefore, we apply a coarse mesh division, as shown in Fig. 6(b) with a mesh size at least ten times that of the microstructure.

Magnetic hysteresis is treated by an energy magnetic hysteresis model [10], [11]. Numerical calculations are performed by the finite element method on a static magnetic field with no eddy or displacement currents.

TABLE III	
COMPARISON OF LOSSES IN THE MICRO-MODEL AND MACRO-MODEL	

		Unit
Particle size	4×4×4	mm
Particle pitch(Basic structure)	5×5×5	mm
Electrical conductivity: $\sigma$	$3.77 \times 10^{7}$	S/m
Relative magnetic permeability: $\mu_r$	1	-

# B. Calculation Results

The calculation results of the macro-model are shown in Fig. 7. Only the density distribution of the magnetic flux is shown, because no eddy current is induced. The magnetic flux density distribution is almost uniform in the homogenous structure, reflecting the sameness of the material constants. In our numerical simulations, the calculation times in the micro- and macro-models are about 17 days and 2 days, respectively.

## C. Evaluation and Discussion

Equations (1) and (2) describe the average magnetic property of the macro-model. Fig. 8 plots the relation between the average magnetic flux density and the average magnetic field in the macro-model and (for comparison) the micromodel. The magnetic properties are almost consistent between the models. The discrepancy between the macro- and microhysteresis curves is likely caused by numerical errors related to the convergence conditions. It is thought that the shape of hysteresis curve in the macro-model may be in agreement with that in the micro-model when convergence criteria, calculation accuracy, and so on are tightened.

To evaluate the availability of our macro-model, we now compare the electromagnetic losses in the macro- and micromodels. The loss in the micro-model is considered to result from eddy currents. Therefore, we calculate the Joules loss as follows:

$$Q_{\text{micro}} = \sum_{\text{BasicStructure}} \frac{\|\dot{j}_{\text{eddy},i}\|^2}{\sigma} \Delta V_i.$$
(3)

The loss in the macro-model is considered to be derived from the magnetic hysteresis property. Therefore, we calculate the magnetic energy as follows:

$$Q_{\text{macro}} = \sum_{\text{HonogeneousStructure}} \oint B d H \Delta V_i.$$
 (4)

The losses in the macro- and micro-models are compared in Table III. The losses are highly consistent.

By using a complex permeability [12], it may be possible to express the local eddy current problems and then to reduce the calculation time. However, in case of magnetic saturation property (magnetic nonlinearity) in magnetic materials, the complex permeability cannot be applied to the macro-model. On the other hands, the equivalent magnetic hysteresis curve in our numerical simulations allows us to express the magnetic nonlinearity in the macro-model calculation.

### **IV. CONCLUSION**

This paper investigated the equivalent magnetic hysteresis (B-H) curve in a magnetic multiscale problem of a complex material composed of electrically insulated conductors. The conventional methods, which calculate the equivalent material constants, could not properly express the local eddy current losses and the extra magnetic field derived from the local eddy current. To solve this problem, we constructed the magnetic hysteresis curve from the volume-averaged magnetic flux density and the surface magnetic field calculated in the micro-model, and applied it to the macro-model. The mesh number of this macro-model is reduced to 1/10 or less than that of the microstructure. Magnetic hysteresis was treated by a free-energy magnetic hysteresis model, and electrical conductivity was ignored in the macro-model. The magnetic hysteresis curve and the numerically calculated losses were consistent in the macro- and micro-models. The proposed method well expressed the local eddy current phenomena.

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#### REFERENCES

- K. Fujisaki and S. Satou, "Angle difference between B vector and H vector in anisotropic electrical steel," *IEEE Trans. Magn.*, vol. 44, no. 11, pp. 3161–3164, Nov. 2008.
- [2] R. Ortiz, M. Leibl, J. W. Kolar, and O. Apeldoorn, "Medium frequency transformers for solid-state-transformer applications—Design and experimental verification," in *Proc. IEEE 10th Int. Conf. Power Electron. Drive Syst.*, Apr. 2013, pp. 1285–1290.
- [3] J. Gyselinck, R. V. Sabariego, and P. Dular, "A nonlinear time-domain homogenization technique for laminated iron cores in three-dimensional finite-element models," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 763–766, Apr. 2006.
- [4] A. Bermudez, D. Gomez, and P. Salgado, "Eddy-current losses in laminated cores and the computation of an equivalent conductivity," *IEEE Trans. Magn.*, vol. 44, no. 12, pp. 4730–4738, Dec. 2008.
- [5] N. Hihat, E. Napieralska-Juszczak, J.-P. Lecointe, J. K. Sykulski, and K. Komeza, "Equivalent permeability of step-lap joints of transformer cores: Computational and experimental considerations," *IEEE Trans. Magn.*, vol. 47, no. 1, pp. 244–251, Jan. 2011.
- [6] M. Liu, O. Hubert, X. Mininger, F. Bouillault, and L. Bernard, "Homogenized magnetoelastic behavior model for the computation of strain due to magnetostriction in transformers," *IEEE Trans. Magn.*, vol. 52, no. 2, pp. 1–12, Feb. 2016.
- [7] K. Fujisaki, M. Fujikura, J. Mino, and S. Satou, "3D Magnetic field numerical calculation by equivalent *B-H* method for magnetic field mitigation," *IEEE Trans. Magn.*, vol. 46, no. 5, pp. 1147–1153, May 2010.
- [8] K. Fujisaki and T. Ikeda, "Equivalent electromagnetic constants for microwave application to composite materials for the multi-scale problem," *Materials*, vol. 6, no. 11, pp. 5367–5381, 2013.
- [9] K. Terada and N. Kikuchi, *Introduction to Equivalent Method*, (in Japanese). Tokyo, Japan: Maruzen, 2003.
- [10] F. Ikeda, "The hysteresis model using free energy," in Proc. Papers Tech. Meeting Static Appl. Rotating Mach. (IEEJ), 2013, pp. 57–62.
- [11] S. Odawara, K. Fujisaki, and F. Ikeda, "Proposing a numerical method for evaluating the effects of both magnetic properties and power semiconductor properties under inverter excitation," *IEEE Trans. Magn.*, vol. 50, no. 11, Nov. 2014, Art. no. 7201004.
- [12] M. Getzlaff, Fundamentals of Magnetism. Berlin; Germany: Springer, 2008.