

Impact of the Inverter DC Bus voltage on the Iron Losses of a Permanent Magnet Synchronous Motor at Constant Speed

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In this paper, the impact of the DC bus voltage on the iron losses of a permanent magnet synchronous motor (PMSM) fed by a pulse-width modulation inverter is evaluated. Under constant speed operation, the modulation index decreases when the DC bus voltage increases. This modifies the harmonic content of the PMSM input voltage and then affects the motor iron losses. The experimental measurements results show that, for the conditions considered in this paper, the total iron losses are 1.8 times larger at the lowest modulation index than at the highest one.

Keywords: inverter, iron loss, magnetic material, permanent magnet synchronous motor

1. Introduction

Many researches demonstrate that the output voltage of a pulse-width modulation (PWM) inverter fed to any wound soft magnetic material tends to increase its iron losses compared to the case of sinusoidal voltage fed operation. Early literature assumes that the high frequency harmonic components in the PWM supplied voltage are responsible for extra eddy current losses only, implying that the hysteresis losses are only slightly impacted⁽¹⁾. More thorough investigations show that high frequency magnetic flux density occurs inside the magnetic material, which is responsible for a skin effect that has a strong impact on the material iron losses⁽²⁾. Moreover, some researchers point out that those high frequency magnetic flux density variations can also cause minor hysteresis loops which increase the hysteresis losses in a sometimes non-negligible way⁽³⁾.

PWM gate signal generation has several parameters which can influence the iron losses in their own way. The impact of the modulation index in particular, has been studied on wound laminated ring cores⁽⁴⁾. The authors conclude that the material specific iron losses tend to increase when the modulation index decreases. One of the cause of this increase is that reducing the modulation index has for effect to widen the minor hysteresis loops in the material⁽⁵⁾.

Under the assumption that the harmonic content of the PWM supplied voltage is only responsible for extra eddy current losses, some authors are able to propose analytical formulas of the iron losses caused by PWM in transformers⁽⁶⁾. As a result, they propose an analytical characterization of the impact of the PWM modulation index on the iron losses.

As for the permanent magnet synchronous motors (PMSM), numerical analysis allows evaluating the iron losses

caused solely by PWM⁽⁷⁾. The physical repartition of these losses in the motor parts (rotor, stator and magnets) can also be evaluated by numerical analysis⁽⁸⁾. In particular, authors conclude that the magnet losses are almost only due to the eddy current caused by the PWM and that the effect of stator slots on the magnet losses is comparatively small. The additional iron losses caused by PWM can also be approximated using analytical formulas⁽⁹⁾⁽¹⁰⁾. In (9), the study focuses on a synchronous motor with surface-mounted permanent magnets (SPMSM). Both experiment and numerical calculation show that the motor iron losses decrease when the modulation index increases. However, the derivative is not considered in the motor equation used for the analytical model, even though the derivative has a significant impact on the current high order harmonics. In (10), the increasing rate of eddy current losses due to PWM is evaluated on a synchronous motor with interior permanent magnets (IPMSM). The motor uses ferrite magnets with high resistivity, which allows neglecting the magnet eddy current losses. Both the analytical calculation and the experiment show lower eddy current losses increasing rate at high modulation index. However, the high frequency content of the motor current due to the PWM carrier signal is considered using an rms value that depends on the PWM control parameters. Then the analytical model does not consider each harmonic of the voltage and current.

In this paper, an experimental methodology to evaluate the influence of the PWM modulation index on a synchronous motor with interior sintered NdFeB magnets is proposed. An analytical model is then developed to give the motor voltage and current harmonics as a function of the PWM control parameters, and especially the modulation index. In section 2, the motor geometry and the cores material characteristics are presented. The section 3 explains the experimental methodology and provides the IPMSM total iron losses measurement results. The experiments are conducted at constant speed and the DC bus voltage is changed so that a fairly wide range of modulation index is covered. The section 4 develops an

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Table 1. Material characteristics

Characteristics	Stator and rotor	Magnet
Material	35H300	NdFeB
Thickness (μm)	350	-
Magnetic saturation (T)	2.12	-
Electrical resistivity ($\mu\Omega\cdot\text{m}$)	0.52	1.6
Residual magnetization (T)	-	1.28

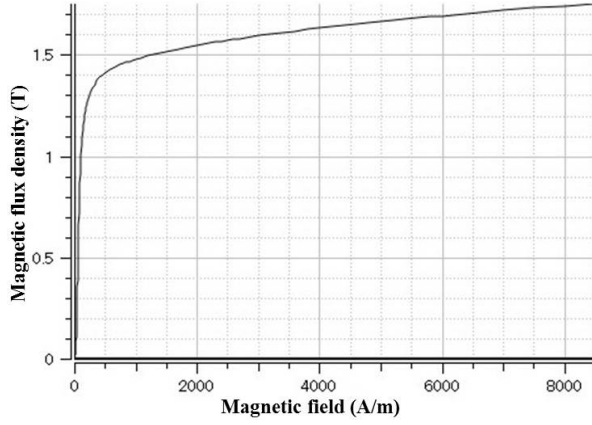
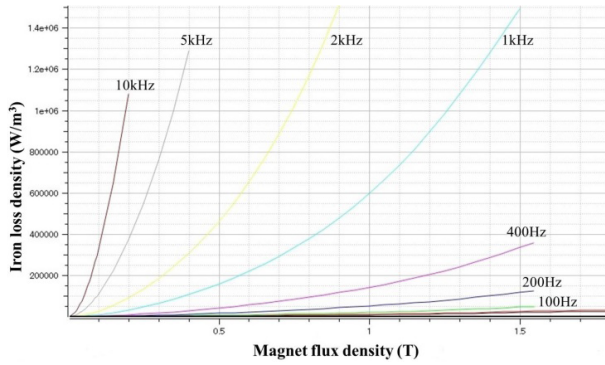

 Fig. 1. B - H characteristic of the 35H300 material


Fig. 2. Specific iron loss density of the 35H300 material

analytical model of the IPMSM voltage and current harmonics as functions of the PWM control parameters. Finally, measurements of the magnetic flux density in the stator are shown in section 5.

2. Material Characteristics and Motor Geometry

2.1 Material Characteristics The stator and rotor of the proposed IPMSM are made of non-oriented electrical steel sheets 35H300. The permanent magnets are sintered NdFeB. The important characteristics of the materials are shown in Table 1. The B - H and specific iron loss characteristics of the soft-magnetic material are given in Fig. 1 and Fig. 2 respectively.

2.2 Motor Geometry The cross-section view of the motor is illustrated in Fig. 3. The geometry characteristics and the motor electrical characteristics are listed in Table 2 and Table 3 respectively.

3. Experimental Methodology and Measurement Results

3.1 Experimental Methodology The experimental test bench is illustrated in Fig. 4. The IPMSM is driven by

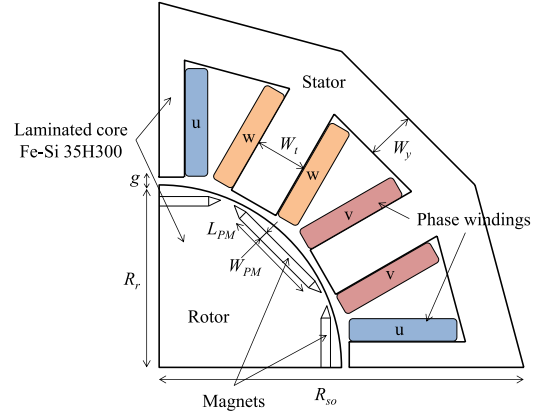


Fig. 3. Quarter cross-section view of the IPMSM

Table 2. IPMSM geometry characteristics

Poles / Slot number	8/12
Radius of stator core R_{so}	64 mm
Radius of rotor core R_r	37 mm
Air gap g	1.25 mm
Yoke width W_y	9.2 mm
Tooth width W_t	10 mm
PM length L_{PM}	20 mm
PM thickness W_{PM}	2 mm
Core length L_c	47 mm
Number of winding turns	37

Table 3. IPMSM electrical characteristics

Phase resistance R_s	0.498 Ω
d-axis inductance L_d	2.44 mH
q-axis inductance L_q	3.70 mH
Peak line to neutral back EMF at 750rpm	20.1 V

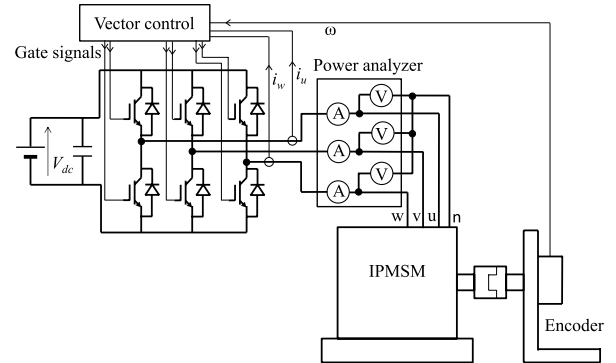


Fig. 4. Experimental test bench for the iron loss measurement

a three-phase two-level voltage source inverter (VSI) with IGBTs. The inverter input voltage V_{dc} is provided by a DC power source. A standard vector control using a fixed frequency PWM with a triangular wave carrier signal is implemented for the speed control. The d-axis current reference is set to zero in the vector control scheme. No load is connected to the IPMSM rotor shaft but an encoder is still needed for the feedback of the shaft rotational speed.

The IPMSM input active electrical power $P_{3\phi}$ is measured using a power analyzer with a sampling frequency of 2.2 MHz and a bandwidth from DC to 2 MHz. On the

measurement window, the power analyzer performs the calculation

$$P_{3\Phi} = \frac{1}{N} \sum_{k=0}^{N-1} \left(i_u(k)v_{un}(k) + i_v(k)v_{vn}(k) + i_w(k)v_{wn}(k) \right) \dots \dots \dots (1)$$

where N is the sample number of the measurement window, $i_{u,v,w}$ are respectively the u , v and w phase currents, and $v_{u,v,wn}$ are respectively the u , v and w phase to neutral voltages. In order to calculate the copper losses, the rms current is obtained from the current waveform using the equation

$$I_{xrms} = \sqrt{\frac{1}{N} \sum_{k=0}^{N-1} i_x^2(k)} \dots \dots \dots (2)$$

where x can be u , v or w depending in the considered phase.

The inverter-fed IPMSM iron losses under no-load condition can then be calculated using the following equation

$$P_{i_nol} = P_{3\Phi} - R_s(I_{urms}^2 + I_{vrms}^2 + I_{wrms}^2) - P_f \dots \dots \dots (3)$$

where P_f are the mechanical losses of the IPMSM and the encoder that are measured beforehand using a rotor with demagnetized magnets. With this equation, it is assumed that the measured iron losses are the sum of the hysteresis losses and the eddy current losses in the stator core, rotor core and magnets.

In order to get an approximation of the impact of the PWM on the total iron losses by experiment, a test at no-current is done. The methodology of this test is explained in details in (11). During this test, an auxiliary brushless DC motor is used to rotate the IPMSM externally. The three phases of the IPMSM stator are in open circuit. Therefore, no current is flowing through the stator windings and the iron losses are caused by the rotating magnets only. In this condition, the IPMSM total iron losses is equal to P_{i_noc} and the contribution of the PWM to the total iron losses is estimated using the simple equation

$$P_{i_PWM} = P_{i_nol} - P_{i_noc} \dots \dots \dots (4)$$

3.2 Iron Loss Measurement Results The PWM carrier frequency is 1 kHz and the inverter dead-time 3500 ns. In order to evaluate the influence of the PWM modulation index on the iron losses, the measurements are made at a constant speed of 750 rpm (50 Hz) with different values of V_{dc} . When changing V_{dc} , the speed control loop modifies the value of the modulation index in order to adjust the inverter output voltage so that the speed is maintained constant. The values of V_{dc} range from 180 V to 43 V, which corresponds to a variation of modulation index m_a from 0.230 to 0.975.

The Fig. 5 shows the evolution of the DC bus voltage, the IPMSM total iron losses under no-current condition P_{i_noc} , no-load condition P_{i_nol} and the IPMSM iron losses caused by the PWM P_{i_PWM} . P_{i_noc} does not depend on the modulation index because the IPMSM is not driven by the inverter in no-current condition. At a modulation index m_a of 0.230, the PWM carrier frequency is responsible for as much as 66% of the total iron losses. At a modulation index of 0.975, this ratio becomes 37%.

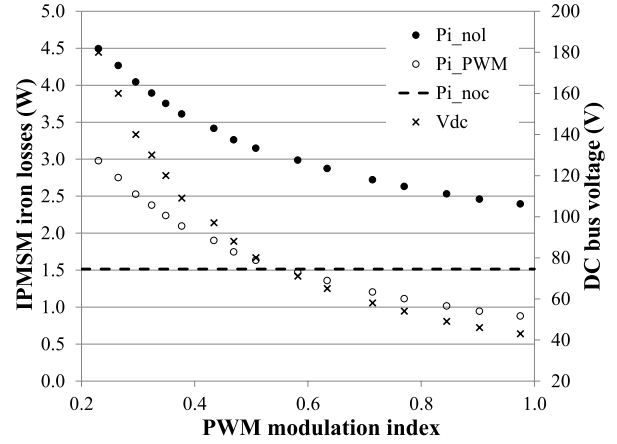


Fig. 5. IPMSM iron loss measurement

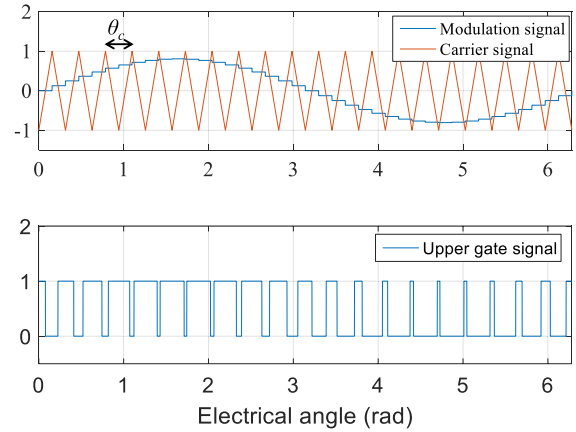


Fig. 6. PWM sampled modulation signal and triangular carrier signal (upper), and IGBT upper gate signal (bottom). Example for u phase with $m_a = 0.8$, $F_c = 1$ kHz and $F_e = 50$ Hz

4. Analytical Model of Motor Voltage and Current

4.1 Calculation of Motor Phase Voltage Harmonics

As explained section 3, the PWM uses a triangular wave shape of frequency F_c and a sinusoidal modulation signal whose amplitude is given by the modulation index m_a and frequency is equal to the IPMSM electrical frequency F_e . The modulation signal is sampled at two times the carrier frequency, which gives the IGBT gate signal illustrated in Fig. 6. The switch OFF points $\theta_{k,off,u}$ and switch ON points $\theta_{k,on,u}$ (in rad) of the u phase upper gate signal can be calculated as a function of the electrical frequency, the carrier frequency and the modulation index⁽⁶⁾. They are given by

$$\theta_{k,off,u} = \frac{\theta_c}{4} (m_a \sin((k-1)\theta_c) - 3 + 4k) \dots \dots \dots (5)$$

$$\theta_{k,on,u} = -\frac{\theta_c}{4} \left(m_a \sin\left(\left(k - \frac{1}{2}\right)\theta_c\right) + 1 - 4k \right) \dots \dots \dots (6)$$

where θ_c is equal to $2\pi F_e/F_c$, k is an integer from 1 to m_c and m_c is equal to F_c/F_e . The switch OFF and switch ON points of the two other phases can be obtained by a simple $2\pi/3$ phase shift in (5) and (6).

By performing a Fourier transformation, it is possible to obtain the Fourier series of the u phase upper gate signal. It

follows

$$C_u(\theta) = \frac{1}{2} + \sum_{n=1}^{\infty} (a_n(C_u)\cos(n\theta) + b_n(C_u)\sin(n\theta)) \quad \dots\dots\dots (7)$$

where n is the harmonic order, θ is the electrical angle (between 0 and 2π) and $a_n(C_u)$ and $b_n(C_u)$ are the Fourier coefficients of the signal. The coefficients depend on the switching times obtained in (5) and (6) and are given by

$$a_n(C_u) = \frac{1}{\pi n} \sum_{k=1}^{m_c} (\sin(n\theta_{k,off,u}) - \sin(n\theta_{k,on,u})) \quad \dots\dots\dots (8)$$

$$b_n(C_u) = -\frac{1}{\pi n} \sum_{k=1}^{m_c} (\cos(n\theta_{k,off,u}) - \cos(n\theta_{k,on,u})) \quad \dots\dots\dots (9)$$

The Fourier series of motor phase to neutral voltage can then be obtained from (8) and (9) using

$$\begin{pmatrix} V_{un}(\theta) \\ V_{vn}(\theta) \\ V_{wn}(\theta) \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} C_u(\theta) \\ C_v(\theta) \\ C_w(\theta) \end{pmatrix} \quad \dots\dots\dots (10)$$

The voltage harmonics amplitudes obtained using the developed model are presented in Fig. 7 and Fig. 8 for two values of m_a . The harmonics are compared with the FFT performed on the voltage waveforms obtained by measurement. It can be seen that the model gives results relatively close to the experimental data. Moreover, the fundamental remains the same for both modulation index values because the rotational speed is constant. Consequently, only the high order harmonics change. Knowing the harmonics amplitudes, calculating the voltage total harmonic distortion (THD) is quite straightforward. The THD values obtained from experimental measurements and model calculation are compared in Table 4.

4.2 Calculation of Motor Phase Current Harmonics

As explained in the previous section, the IPMSM rotates without load. Consequently, the fundamental of the phase current can be considered null as an approximation. This means that the fundamental of the phase voltage and the fundamental of the IPMSM back-EMF have no phase or amplitude difference. According to a preliminary measurement, the back-EMF of the u phase can be approximated using the following equation.

$$e_u(\theta) = 2\pi F_e (\Psi_{f1} \sin(\theta) + \Psi_{f5} \sin(5\theta + \phi_{f5}) + \Psi_{f7} \sin(7\theta + \phi_{f7})) \quad \dots\dots\dots (11)$$

where Ψ_{f1} is equal to 0.0641 V.s/rad, Ψ_{f5} is equal to 0.00232 V.s/rad, Ψ_{f7} is equal to 0.00440 V.s/rad, and ϕ_{f5} and ϕ_{f7} are almost equal to $-\pi$. Moreover, given the known relation between the inverter output voltage fundamental, V_{dc} and m_a , it is possible to write

$$2\pi F_e \Psi_{f1} = m_a \frac{V_{dc}}{2} \quad \dots\dots\dots (12)$$

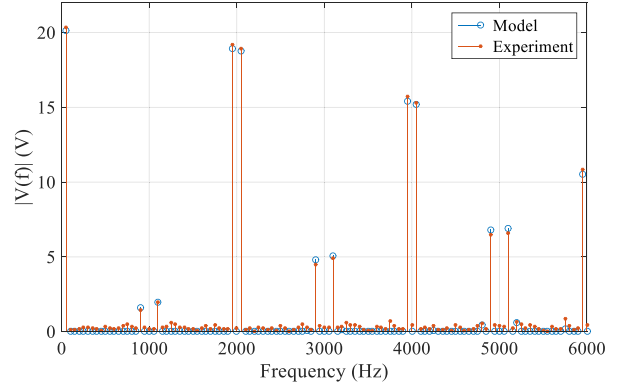


Fig. 7. Phase voltage harmonics amplitudes for $m_a = 0.230$, $F_c = 1$ kHz and $F_e = 50$ Hz

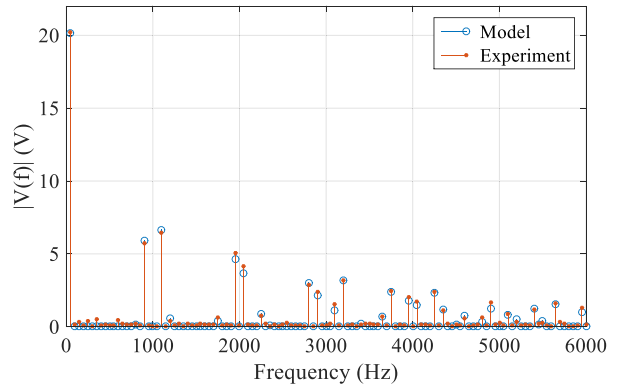


Fig. 8. Phase voltage harmonics amplitudes for $m_a = 0.975$, $F_c = 1$ kHz and $F_e = 50$ Hz

Table 4. Voltage THD comparison

	Model	Experiment
$m_a = 0.230$	232 %	221 %
$m_a = 0.975$	71.5 %	72.5 %

In order to find an analytical expression of the phase current harmonics, it is proposed to perform a Clarke-Park transformation on the voltage expression found in (10) in order to obtain their expression in the rotating (d, q) frame. By first performing the Clarke transformation, the phase voltage in the (α, β) frame is equal to

$$\begin{pmatrix} V_\alpha(\theta) \\ V_\beta(\theta) \end{pmatrix} = \sum_{n=1}^{\infty} \begin{pmatrix} a_n(V_\alpha) \cos(n\theta) + b_n(V_\alpha) \sin(n\theta) \\ a_n(V_\beta) \cos(n\theta) + b_n(V_\beta) \sin(n\theta) \end{pmatrix} \quad \dots\dots\dots (13)$$

with

$$a_n(V_\alpha) = \sqrt{\frac{2}{3}} \left(a_n(V_u) - \frac{1}{2} a_n(V_v) - \frac{1}{2} a_n(V_w) \right) \quad \dots\dots (14)$$

$$b_n(V_\alpha) = \sqrt{\frac{2}{3}} \left(b_n(V_u) - \frac{1}{2} b_n(V_v) - \frac{1}{2} b_n(V_w) \right) \quad \dots\dots (15)$$

$$a_n(V_\beta) = \frac{1}{\sqrt{2}} (a_n(V_v) - a_n(V_w)) \quad \dots\dots\dots (16)$$

$$b_n(V_\beta) = \frac{1}{\sqrt{2}} (b_n(V_v) - b_n(V_w)) \quad \dots\dots\dots (17)$$

where $a_n(V_u)$, $a_n(V_v)$, $a_n(V_w)$, $b_n(V_u)$, $b_n(V_v)$ and $b_n(V_w)$ are the Fourier series coefficients of the phase voltage in the (u ,

v , w) frame calculated using (10). Then by performing the Park transformation, the phase voltage in the (d, q) frame is equal to

$$\begin{pmatrix} V_d(\theta) \\ V_q(\theta) \end{pmatrix} = \begin{pmatrix} a_0(V_d) \\ a_0(V_q) \end{pmatrix} + \sum_{n=1}^{\infty} \begin{pmatrix} a_n(V_d) \cos(n\theta) + b_n(V_d) \sin(n\theta) \\ a_n(V_q) \cos(n\theta) + b_n(V_q) \sin(n\theta) \end{pmatrix} \dots (18)$$

with

$$a_0(V_d) = \frac{1}{2}(a_1(V_\alpha) + b_1(V_\beta)) \dots (19)$$

$$a_0(V_q) = \frac{1}{2}(a_1(V_\beta) - b_1(V_\alpha)) \dots (20)$$

$$a_n(V_d) = \frac{1}{2}(a_{n+1}(V_\alpha) + b_{n+1}(V_\beta) + a_{n-1}(V_\alpha) - b_{n-1}(V_\beta)) \dots (21)$$

$$b_n(V_d) = \frac{1}{2}(b_{n+1}(V_\alpha) - a_{n+1}(V_\beta) + b_{n-1}(V_\alpha) + a_{n-1}(V_\beta)) \dots (22)$$

$$a_n(V_q) = \frac{1}{2}(a_{n+1}(V_\beta) - b_{n+1}(V_\alpha) + b_{n-1}(V_\alpha) + a_{n-1}(V_\beta)) \dots (23)$$

$$b_n(V_q) = \frac{1}{2}(b_{n+1}(V_\beta) + a_{n+1}(V_\alpha) + b_{n-1}(V_\beta) - a_{n-1}(V_\alpha)) \dots (24)$$

It should be noted that when $n = 1$ in (21) to (24), $a_0(V_\alpha)$, $b_0(V_\alpha)$, $a_0(V_\beta)$ and $b_0(V_\beta)$ are null.

After performing the same transformation on the back-EMF, it is possible to write the phase current in the (d, q) frame by using the IPMSM dynamic equation

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} I_d(\theta) \\ I_q(\theta) \end{pmatrix} &= \begin{pmatrix} -\frac{R_s}{L_d} & \frac{2\pi F_e L_q}{L_d} \\ -\frac{2\pi F_e L_d}{L_q} & -\frac{R_s}{L_q} \end{pmatrix} \begin{pmatrix} I_d(\theta) \\ I_q(\theta) \end{pmatrix} \\ &+ \begin{pmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{pmatrix} \begin{pmatrix} V_d(\theta) - e_d(\theta) \\ V_q(\theta) - e_q(\theta) \end{pmatrix} \dots (25) \end{aligned}$$

Since it is quite difficult to derive the expressions of the phase currents directly from (25), the following assumption is proposed. Since the fundamental of the phase currents have been taken null in the previous assumption, this means that the DC components of I_d and I_q are also null. Consequently, I_d and I_q only have high order harmonics due to the PWM and also a 6th harmonic due to the back-EMF. It is then thought that (25) can be approximated to its expression at high frequency.

$$\frac{d}{dt} \begin{pmatrix} I_d(\theta) \\ I_q(\theta) \end{pmatrix} = \begin{pmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \end{pmatrix} \begin{pmatrix} V_d(\theta) - e_d(\theta) \\ V_q(\theta) - e_q(\theta) \end{pmatrix} \dots (26)$$

In this way, the phase currents can be written

$$\begin{pmatrix} I_d(\theta) \\ I_q(\theta) \end{pmatrix} = \sum_{n=1}^{\infty} \begin{pmatrix} a_n(I_d) \cos(n\theta) + b_n(I_d) \sin(n\theta) \\ a_n(I_q) \cos(n\theta) + b_n(I_q) \sin(n\theta) \end{pmatrix} \dots (27)$$

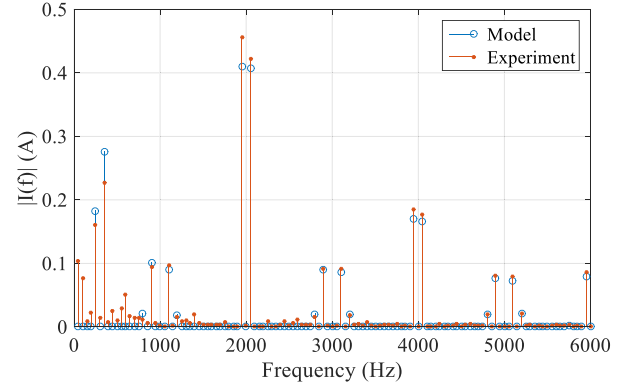


Fig. 9. Phase current harmonics amplitudes for $V_{dc} = 180$ V, $F_c = 1$ kHz, $F_e = 50$ Hz, $m_a(\text{model}) = 0.224$ and $m_a(\text{exp}) = 0.230$

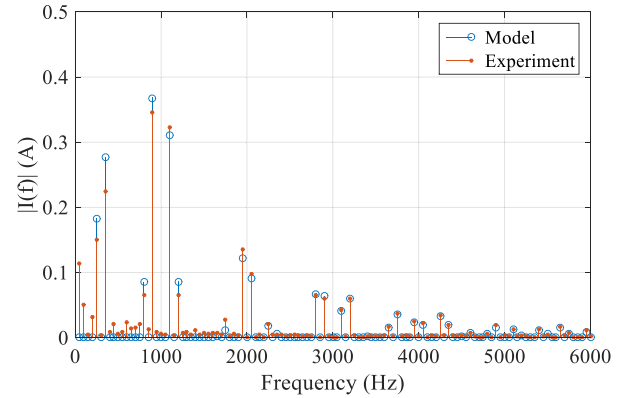


Fig. 10. Phase current harmonics amplitudes for $V_{dc} = 43$ V, $F_c = 1$ kHz, $F_e = 50$ Hz, $m_a(\text{model}) = 0.937$ and $m_a(\text{exp}) = 0.975$

with

$$a_n(I_d) = \frac{1}{L_d} \left(-\frac{b_n(V_d)}{2\pi n F_e} + \Psi_{dn} \right) \dots (28)$$

$$b_n(I_d) = \frac{a_n(V_d)}{L_d 2\pi n F_e} \dots (29)$$

$$a_n(I_q) = -\frac{b_n(V_q)}{L_q 2\pi n F_e} \dots (30)$$

$$b_n(I_q) = \frac{1}{L_q} \left(-\frac{a_n(V_q)}{2\pi n F_e} + \Psi_{qn} \right) \dots (31)$$

and

$$\Psi_{dn} = \begin{cases} -\sqrt{\frac{3}{2}} \left(\frac{\Psi_{f5} + \Psi_{f7}}{n} \right), & n = 6 \\ 0, & n \neq 6 \end{cases} \dots (32)$$

$$\Psi_{qn} = \begin{cases} \sqrt{\frac{3}{2}} \left(\frac{\Psi_{f5} - \Psi_{f7}}{n} \right), & n = 6 \\ 0, & n \neq 6 \end{cases} \dots (33)$$

Equations (27) to (33) give an analytical form of the phase currents harmonics in the (d, q) frame. Similarly to what was done from (13) to (24), the harmonics in the (u, v, w) frame can be obtained by inverse Clarke-Park transformation. The obtained harmonic amplitudes are illustrated in Fig. 9 and Fig. 10 for $V_{dc} = 180$ V and $V_{dc} = 43$ V respectively. It can be seen that the fundamental component is neglected in the model due to the no-load approximation but, although low,

it appears in the experimental data. This is due to the low electromagnetic torque that is actually needed to balance the iron losses and the mechanical friction losses. Moreover, the 5th and 7th harmonics of the current given by the model are higher than found by experiments. This is thought to be due to the approximation done in (26).

5. Observation of Stator Magnetic Flux Density

In the former sections, an expression of the high order harmonics caused by the PWM control on the IPMSM phase currents was established. Moreover the link between the PWM modulation index and the amplitude of these harmonics has been clarified. Those high order harmonics are responsible for magnetomotive forces originating from the stator windings and varying at high frequencies. These magnetomotive forces are responsible for high frequency variations of the magnetic flux density in the rotor, stator and magnets of the IPMSM. Ideally, the analysis should include an analytical calculation model of the magnetic flux density using a simplified linear magnetic model. However, this study is left for future work and in this section, measurements of the magnetic flux density in the stator is shown.

Besides the three-phase armature windings, additional coils are also wound on the stator core teeth and yoke parts in order to measure the average magnetic flux density. The coil has $N_B = 10$ turns and delimits an open surface S which is either the yoke or teeth surface. While the motor is rotating, the voltage V_b induced in the coil is measured at a sampling frequency of 1 MHz. This voltage can be used to estimate the average magnetic flux density through the teeth or yoke surface using the equation

$$B(t) = \frac{1}{N_B S} \int_0^t V_b(u) du \dots \dots \dots (34)$$

The measurements have been done at V_{dc} equal to 43 V and 180 V and are illustrated in Fig. 11 and Fig. 12 for the teeth

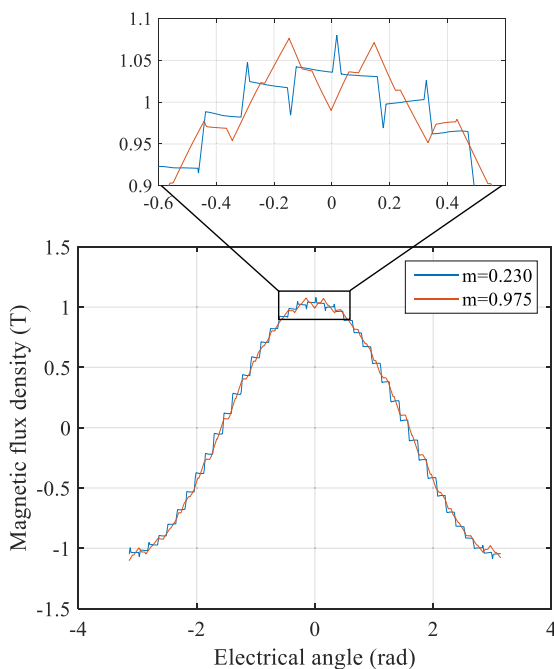


Fig. 11. Teeth magnetic flux density

and yoke respectively. On the scale of the carrier frequency, the modulation index has a noticeable influence on the magnetic flux density waveform. Fig. 13 illustrates the FFT analysis on the teeth magnetic flux density. The fundamental stays unchanged because it is mainly due to the magnet rotation. Moreover, similarly to what is observed on the phase currents, the harmonics caused by the PWM control depend on the modulation index. At high modulation index, the strongest harmonics are around 1 kHz while at low modulation index, the strongest harmonics occurs around 2 kHz and 4 kHz. As in the voltage and current, the high order harmonics are stronger in case of low modulation index. In order to evaluate this phenomenon quantitatively, Table 6 shows the comparison between the low order harmonic contents and the high order harmonic contents. These contents are basically the rms value of the magnetic flux density harmonics in a given frequency range. It can be seen that at low modulation index, the high order harmonic content increases by

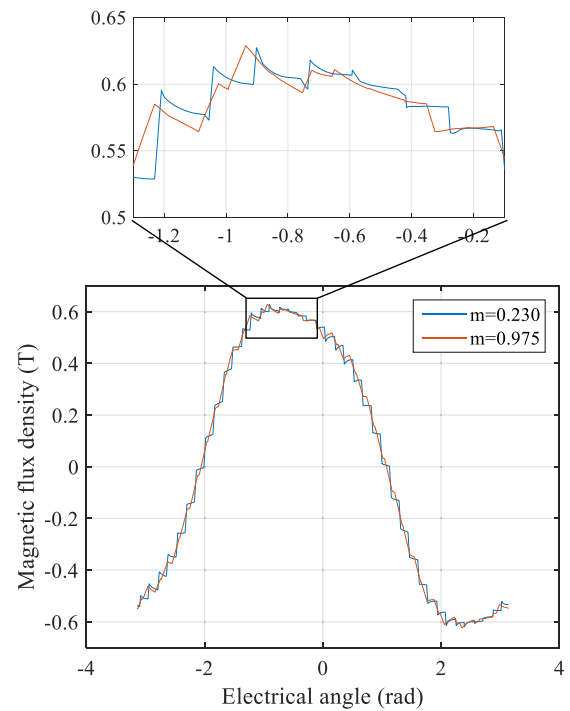


Fig. 12. Yoke magnetic flux density

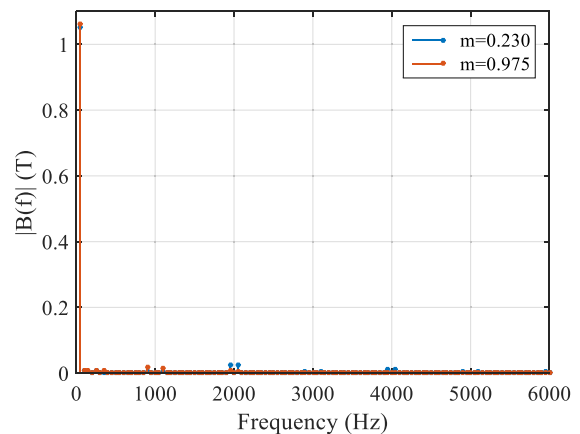


Fig. 13. FFT of the magnetic flux density measured on the teeth

Table 5. Harmonic content comparison of teeth magnetic flux density

	Low order harmonic content (100 to 1200 Hz)	High order harmonic content (1.25 to 51.2 kHz)
$m = 0.230$, Vdc = 180 V	0.0108 T	0.0410 T
$m = 0.975$, Vdc = 43 V	0.0262 T	0.0112 T
Increase rate	- 58.9 %	+ 267 %

more than 250% compared the high modulation index.

6. Conclusion

This paper proposes an experimental evaluation of the dependence between an IPMSM iron losses and the PWM modulation index used for inverter control. It is demonstrated that in no-load condition, the iron losses at a modulation index of 0.230 are 1.8 times higher than at a modulation index of 0.975. An analytical model that relates the PWM control parameters to the phase voltage and current harmonics has been developed and its validity was confirmed by comparison with experimental data. Using this model, the impact of the modulation index on the high order harmonics of the phase currents has been clarified. It is concluded that low modulation index increases the high order harmonic content of the phase current, which in turn increase the high order harmonic content of the magnetic flux density in the motor, then increasing the iron losses. In future works, an analytical linear magnetic model will be developed in order to relate the phase current harmonics to the high frequency magnetic flux density variations and then to the iron losses. Overall, the aim will be to relate the PWM modulation index to the iron losses of each part of the IPMSM.

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