Mathematics and Dialectic in Plato's *Republic* VI-VII[1]

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1. Introduction

In a previous paper on the Simile of the Line in Plato's *Republic* Book VI, I have argued that the visible (*horaton*) represents the sensible in general and the opinable, that *eikasia* means taking an image for its original, that the equality of the middle two segments (DC and CE) of the line is an unintended consequence of the mechanism of the Simile of the Line, and that the objects of *dianoia* are mathematical intermediates (Asano 1997A).[2] If my arguments there are accepted, they will support a traditional view of the Simile, according to which the Simile first aims to illustrate the relation between the visible and the intelligible by the relation between images and their originals, and then to distinguish mathematical intermediates from Forms within the intelligible objects.[3]

A major, related issue that was not discussed in my previous paper (1997A) is how mathematics and dialectic are different in their methods. We know that they are concerned with different kinds of objects, intermediates and Forms; however, it is mostly the differences in methods that Plato tells us about mathematics and dialectic in the Simile of the Line and the subsequent pages of Book VII. On this ground some interpreters have tried to eliminate the difference of objects between mathematics and dialectic in favor of the differences in methods.[4] So I want to discuss in this paper the

[1] This paper is originally written as a chapter of my Ph. D. dissertation submitted to the University of Texas at Austin in December, 1997.

[2] Since the present paper is a sequel to the previous paper, please read Asano 1997A, too.

[3] For a traditional view of the Simile of the Line, see Hardie: 49–65.

mathematicians' and philosophers' methods as understood by Plato in the *Republic*.[5] In the rest of the introduction, I shall outline what Plato says about the differences between mathematics and dialectic, and the problems involved in it; after that I shall discuss those problems in the following sections.

In the Simile of the Line, Plato tells us that mathematics and dialectic differ in two ways, in these words:

> in one part of it [mathematics] a soul, using as images the things that were previously imitated, is compelled to investigate on the basis of hypotheses and makes its way not to a beginning but to an end; while in the other part [dialectic] it makes its way to a beginning that is free from hypotheses; starting out from hypothesis and without the images used in the other part, by means of forms themselves it makes its inquiry through them.[6] (*Republic* 510b4–9)

The first feature of mathematics here stated is its use of images, and what it uses as images is "the things that were previously imitated." The second feature is that mathematics investigates from hypotheses to an end. By contrast dialectic does not use images, and it investigates from hypotheses to a beginning. What Plato means by those differences is still quite unclear, as indicated by Glaucon's response (510b10).[7] There are four major, overlapping problems. First what are the hypotheses, and how are they used? Second what does it mean to investigate from hypotheses to an end? or to a beginning? Third what does it mean for mathematics to use images, and why does it do so? Fourth why is it mathematics that is chosen here as a prelude to dialectic? These

4 Cf. Robinson: 192–201; N. Cooper: 65–9; and N. White 1979: 184–6; 1976: 96–8, 109–11. It would also be possible to include here Cross & Woozley: 230–38. For a criticism of their views, see Asano 1997A.

5 Emphasis will be put on the mathematicians' methods.

6 I shall use Bloom's translation of the *Republic* (with minor stylistic changes), unless otherwise noted.

7 Glaucon's words "I don't sufficiently (*ouch hikanôs*) understand" are a rhetorical understatement, which actually means "I don't understand at all." Cf. a genuine case of partial negation "I understand although not adequately (*hikanôs ou*)" which comes after some elucidation of Plato's meaning at 511c3.

four problems I shall discuss in the following sections.

2. The Hypotheses

In this section I shall discuss the problem of hypotheses. About the mathematicians' hypotheses Plato says:

> the men who work in geometry, calculation, and the like hypothesize (*hupothemenoi*) the odd and the even, the figures, three forms of angles, and other things akin to these in each kind of inquiry, and make them hypotheses (*hupotheseis*). (510c2–6)

The examples of hypotheses here are "the odd and the even, the figures, three forms of angles."[8] The word "*hupothesis*"[9] (hypothesis) derives from "*hupotithêmi*"[10] (hypothesize), and a hypothesis is something that is hypothesized. Now "*hupotithêmi*" means: to place under, to lay down.[11] Thence "*hupothesis*" can mean a foundation, or an assumption.[12] So mathematicians lay down the odd and the even, the figures, and three forms of angles, and make them hypotheses. Scholars disagree, however, on what sort of things those hypotheses are. For example, Archer-Hind (102) thinks that a hypothesis is a definition. Robinson and Annas think that it is any kind of proposition.[13] Cornford and Hardie think that it is an existential proposition.[14]

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8 The figures will include the square itself and the diagonal itself (510d7–8), and the one (524e1, 524e6–525a2, 526a2–4) is most likely another example of hypothesis in calculation.

9 The singular form of "*hupotheseis*".

10 The dictionary form of "*hupothemenoi*".

11 Cf. Liddell and Scott.

12 For a detailed study of the origin of the meaning of "*hupotithêmi*" and "*hupothesis*", see Robinson: 93–100. There is an interesting and important ambiguity in the concept of *hupothesis*. Whether something becomes a foundation or an assumption in one's theory depends on one's attitude to it: if one is sure of, and content with, what is laid down, it is a foundation (from one's own point of view); while if one is not sure of, or content with, what is laid down, it is an assumption.

Crombie and Hare think that it is a thing (or concept).[15]

In support of the definitional view of hypotheses, Plato says that calculation asks what the one itself is (524e6–525a2), and he seems to provide the mathematician's definition of the one as something that "contains no parts within itself" and is "each equal to every other one" (526a2–4). But on the other hand, Plato makes it clear that mathematicians do not give an account (*logos*) of their hypotheses (510c6–7, 533c2–3, 534b4–6), and that it is the function of dialectic to try to grasp what each thing is (533b2–3), which is *logos* of its being (534b3–4). This is another important difference between mathematics and dialectic: mathematics does not give an account while dialectic does. This, however, raises a further question: what it is to give an account (*logon didonai*).

To give an account (*logon didonai*) is commonly taken either as giving a definition of something or as giving a proof of something. The view of giving an account as giving a proof will naturally go with the propositional view of hypotheses because what one gives a proof of is a proposition, and not a thing.[16] On the other hand, if giving an account means giving a definition, then the mathematicians *do not* give definitions, while the dialecticians do. This is incompatible with the definitional view of hypotheses, according to which the mathematicians *do* lay down definitions as hypotheses. So whichever meaning it may take to give an account, the definitional view of hypotheses is most likely wrong.[17] That may not be the case, however. For, I suggest,

13 Robinson: 99–101, 152; Annas: 287–90; and also Cross & Woozley: 246–8. C. C. W. Taylor, too, thinks that what is laid down is some sort of proposition, including a definition (194–5, 198–9).

14 Cornford 1932: 65; and Hardie: 60. Also Nettleship: 252; Ross: 51; Gosling 1973: 109–12; and N. White 1976: 98.

15 Crombie, Vol. 1: 113; Hare: 22–4, 27; and also Raven: 154. Guthrie, too, thinks that what is laid down is a thing (509); however, he is a little ambiguous on this problem just as I would like to be (510, 525).

16 Cf. C. C. W. Taylor: 195, 197; and Annas: 287.

17 Perhaps the definitional view can be compatible with the propositional view of

there can be two levels of definitions, mathematically adequate ones and dialectically adequate ones, so that the mathematicians answer the question of what a thing is on one level but not on the other.[18] Although it is very difficult to state exactly what mathematically adequate and dialectically adequate definitions are, it is possible to give a rough idea of what they are. Before we go on to that difficult task, however, let me discuss the propositional view of hypotheses first.

The propositional view of hypotheses is wrong in one sense but right in another. As a general theory of hypotheses, what is laid down in a discussion is surely something of which we can say either that it is true or that it is false.[19] But what Plato actually says is "hypothesize the odd and the even, the figures, three forms of angles, and other things akin to these" (510c3–5), and he does not mention any proposition whatsoever. Scholars have suggested propositions that Plato might have had in mind. For example, A. E. Taylor suggests about the odd and the even, a proposition that all numbers are integers.[20] According to him, Plato believed that the mathematicians' hypothesis that all numbers are integers is false, because there are irrational numbers; and that it can be shown by dialectic. A. E. Taylor's suggestion is based on his understanding (84) that "destroying the hypotheses" at Book VII 533c8 means to disprove the hypotheses. This is another phrase that needs interpretation. In the passage where the phrase is found, Plato says about dialectic:

> only the dialectical way of inquiry proceeds in this direction, destroying the hypotheses (*tâs hupotheseis anairousa*), to the beginning itself in order to make it secure.[21] (533c7–9)

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hypotheses; that is, if the proposition meant is of a form "the definiendum is the definiens."

18 The idea of two levels of understanding is quite common; but to take them as two levels of definitions (accounts of what F is) is not so common. For similar ideas of mathematically (scientifically) adequate and dialectically adequate accounts of F, see Murphy: 174, 178; Crombie, Vol. 1: 118–19, 124–5, 130; and Reeve: 59–60, 72–9.

19 Cf. Robinson: 93–6, 99; and C. C. W. Taylor: 195.

20 A. E. Taylor: 81–2. Cf. Robinson: 103; and Cross & Woozley: 247.

"*Anaireô*"[22] means to take away, especially, to rescind laws and customs;[23] and so "*tâs hupotheseis anairousa*" means to destroy the hypotheses.

There is, however, no evidence that Plato thought the mathematicians' hypotheses to be false.[24] He says that mathematicians "lay hold of something of what is" (533b7) and "dream about what is" (533b8–c1). These expressions are to be taken positively as they point to the superiority of mathematics over the other arts concerned with becoming (533b3–8). Dreaming is, according to Plato, believing an image to be its original (476c6–7). Just as there are three types of image-original relationships, between the objects of *eikasia* and *pistis*, between the objects of opinion and knowledge, and between the objects of mathematics and dialectic, there are three types of dreaming: *eikasia*, opinion, and mathematics. To take a case of opinion, people who believe that to pay back one's debts is just, are dreaming about justice insofar as they do not know the definition of justice. But it does not follow that their opinion is wrong. In the same way, even though mathematicians dream about what is, it does not follow that their hypotheses are false.

Plato also says that "the objects of mathematics are, given a beginning, intelligible" (511d2). So he does not mean that the mathematicians' hypotheses are false. What he means is that the mathematicians do not know what they dream about (511d1–2, 533c1). The reason for this is that they "use hypotheses and, leaving them untouched, are unable to give an account of them" (533c1–3).[25] By contrast, we can

21 It is not clear what the grammatical subject and object of "make it secure (*bebaiôsêtai*)" are. But most likely, the subject is the dialectical way of inquiry, and the middle voice of the verb is the direct reflexive middle, in which case the object is the same as the subject, rather than the indirect reflexive middle, in which case the object would be either the hypotheses or the beginning itself.

22 The dictionary form of "*anairousa*".

23 Cf. Liddell and Scott.

24 I do not mean that Plato thought that mathematicians never made an error, but I believe that he thought that most of the mathematics at his time was genuinely true.

25 More accurately speaking, mathematicians do not know what they dream about,

infer, dialectic destroys the hypotheses by giving an account of them.[26] This means that when dialectic gives an account of the hypotheses, they "lose their hypothetical character."[27]

Another proponent of the propositional view of hypotheses, Robinson, suggests, for example about the three forms of angles, a proposition "that every plane angle is either right or obtuse or acute."[28] Certainly it is perfectly possible that Plato had some such proposition in mind, since there is nothing in the text which excludes that proposition. But there is nothing in the text which leads us to it, either.

Where there is no hint as to what specific proposition Plato had in mind about the three forms of angles, a more plausible suggestion is an existential proposition that there is a right angle, etc. As C. C. W. Taylor has pointed out, in the *Parmenides* (136a4–c5) Plato seems to regard "as equivalent the expressions *hupotithesthai ti* [hypothesize something] and *hupotithesthai ti einai* [hypothesize that there is something]" (198). Similarly in *Republic* Book VI (507b2–7), Plato seems to regard as equivalent the expressions "setting down (*tithentes*) something" and "assert that there is something". Again when Plato writes in Book X that "we . . . set down (*tithesthai*) some one particular form" (596a6–7), he means that they assert that there is "some one particular form." Indeed, what philosophers do at the beginning of their investigation is "hypothesizing that there is a fair, itself by itself, a good, a large, and all the rest"

because they do not know their hypotheses (533c1–5), which in turn is because they cannot give an account of them (534b4–6).

26 Cornford, who distinguishes mathematical dialectic and moral dialectic, suggests that the former destroys its hypotheses by giving a proof of them while the latter destroys its hypotheses in the sense of amending or abolishing (1932: 86); and he is followed by Guthrie: 525. But I do not think that there are two kinds of dialectic with different procedures, although I acknowledge that there are two branches of dialectic which share the same methods.

27 Cross and Woozley: 248. See also Ross: 57; Robinson: 161–2; Annas: 288; and Reeve: 77.

28 Robinson: 103. He is followed by Cross and Woozley: 247.

(*Phaedo* 100b5–7).[29] Just as philosophers introduce Forms, mathematicians introduce mathematical objects. That is, they assume that there are Forms or mathematical objects. For, unless there are such objects, it would be nonsense to talk about them (cf. Nettleship: 252). So if the mathematician's hypothesis is ever a proposition, it would be an existential proposition affirming the existence of the objects of mathematics.

I have suggested in the last paragraph that Plato regarded as equivalent the expressions "hypothesize something" and "hypothesize that there is something." Granted that they are equivalent, which one is primary for Plato? Plato's use of the construction of *hupotithesthai* with a direct object suggests that what is hypothesized is primarily a thing. This seems quite reasonable; because it is a thing that makes an existential proposition true, and an existential proposition does not make a thing exist. So Crombie's and Hare's view of mathematicians' hypothesis as a thing seems the closest to Plato's view. This, however, excludes neither a propositional view nor a definitional view of hypotheses.[30] My view is that the three elements, a thing, a proposition, and a definition, are all involved in a hypothesis.[31]

First, one cannot hypothesize a thing F without affirming an existential proposition that there is F, at least if one is doing a science about reality. Mathematics as well as dialectic is such a science (*noêsis*)[32] for, according to Plato, the things that are hypothesized by mathematicians as well as dialecticians, are emphatically beings (*onta* or *ousia*), more real than any ordinary things around us.[33] So mathematicians cannot

[29] This is my translation based on Gallop's.

[30] The principal defect of Crombie's and Hare's view is that they do not take seriously the existence of the things hypothesized in mathematics, and that especially Hare (27) tends to equate those things with concepts.

[31] Cf. my argument for the unity of the existential and predicative uses of being in Asano 1994: 21–2.

[32] This "*noêsis*" at 534a2 is a general term that includes both *noêsis* and *dianoia* in the Simile of the Line (511d8).

[33] This is a view which Plato and mathematicians share about the objects of mathematics (cf. 525d5–526a7).

hypothesize, for example, the square without affirming an existential proposition that there is the square. Further, one cannot hypothesize a thing F, nor an existential proposition that there is F, without having any idea of what F is. So mathematicians have to have some idea of what those objects are which they hypothesize; otherwise, they could not know what in the world they are hypothesizing.[34] It is connected with Plato's and mathematicians' interest. Suppose that mathematicians hypothesize the square or that there is the square. They do not just hypothesize the existence of a name "the square". What Plato and mathematicians are interested in, is the thing called the square and its nature. Thus Plato constantly asks the question of what a thing is, and it is asking this question that lifts mathematicians as well as philosophers above the world of opinion (524c10–526b3, 533b1–3).

Now we can go back to the question of what it is to give an account. The kind of account which dialecticians give is an account of what a thing is (533b2–3, 534b3–6). Accordingly, the account which mathematicians fail to give is an account of what their hypotheses are. So "to give an account" means to give a definition of something.[35] Nevertheless, mathematicians must know somehow what their hypotheses are, in order to hypothesize them. That is where my suggestion comes in that there are two levels of definitions.

In another paper (1996) I have argued that there are two arguments for Forms, the Argument from Conflicting Appearances (ACA) and the One over Many Argument (OMA). They reflect the two aspects of the Socratic question "what is F?" or the two ways in which it can be taken. First, it can be taken as what is F and never not-F,[36] and second, as what is that which is common to F things and makes them F.[37] Socrates'

34 Cf. the paradox of inquiry in the *Meno* (80d5–e5).

35 This will give an additional support to the view of a hypothesis as a thing, of which one can, or cannot, give a definition.

36 In this analysis, "what" is the subject and "F" is a predicate.

37 In this analysis, "that" is the subject and "what" is a subjective complement.

interlocutors in Platonic dialogues often take his question in the first way; for example, to the question "what is courage?" Laches answers that "he is a man of courage who does not run away, but remains at his post and fights against his enemy" (*Laches* 190e5–6), and to the question "what is beauty?" Hippias answers that "a beautiful maiden is a beauty" (*Hippias Major* 287e3–4).[38] Although Socrates rejects those answers by saying that they are not the kind of answer he wants, they are not entirely wrong answers, either.[39] For they are attempts to state paradigmatic cases of F (courage and beauty respectively) that the interlocutors believe are F and never not-F.[40]

[38] So do Euthyphro in the *Euthyphro* 5d8–e2, Meno in the *Meno* 71e1–72a5, and Theaetetus in the *Theaetetus* 146c7–d3. The translation of the *Laches* and *Hippias Major* is Jowett's in Hamilton and Cairns.

[39] The passages where Socrates complains or suggests that his interlocutors did not answer his questions are *Laches* 190e8–9, *Hippias Major* 289c3–d5, *Euthyphro* 6d9–11, *Meno* 72a7–8, and *Theaetetus* 146d4–e10.

[40] Laches, Hippias, Euthyphro, and Theaetetus (but perhaps not Meno) are aware that Socrates is asking for not just any cases, but solid and exemplary cases of F: solid in the sense that they are the last things to be not-F (cf. *Protagoras* 330d7–9, *Hippias Major* 288b1–3), and exemplary in the sense that by comparing with them one can determine if other cases are F (cf. *Euthyphro* 6e5–7, and *Republic* 472c4–d7). Thus believing that they have provided such cases of F, they, excepting Theaetetus, hold a strong conviction of the truth of their statements (*Laches* 190e6; *Hippias Major* 287e4, 288a3–5; and *Euthyphro* 6d5).

Nehamas (1975) has suggested a new reading of the answers Socrates' interlocutors give to Socrates' questions in the early dialogues, which is similar to Gosling's view on *ta polla kala* in the *Republic* (for Gosling's view, see Asano 1993: 127-25; and 1994: 17-20). According to Nehamas, Socrates' interlocutors answer the question what is F by telling not concrete examples (particulars) but many accounts of F. I do not agree with him. Certainly it is true that the interlocutors' answers are not always expressed in terms of concrete examples. This is clear in the *Meno* 71e1–72a2 and *Theaetetus* 146c7–d3 where the examples given by Meno and Theaetetus are not concrete examples but they are kinds of virtue and knowledge respectively. (I do not mean that the *Theaetetus* is an early dialogue.) But on the other hand Laches', Hippias' and Euthyphro's answers are naturally read as referring to particular people and actions. Consequently the moral to draw is that it is insignificant whether the examples are concrete or general. Whether the examples are concrete or general, Socrates' complaint is the same that those examples do not answer his questions. Next the interlocutors' answers should better be taken as examples rather than accounts of F. The

This aspect of the Socratic question is captured by ACA, which infers from the premise that perceptible objects which appear F also appear not F, the conclusion that there must be an F itself which is incapable of appearing not F.[41] Mathematical objects (intermediates) are what is generated by this argument in *Republic* Book VII (523b9–526a7).[42] Thus the mathematicians who hypothesize mathematical objects[43] answer the Socratic question "what is F?" in one way.[44]

Second, Socrates in Platonic dialogues most often explains his question, focusing on its second aspect. He says, for example, to Laches, "what is that common thing which is the same in all these cases of courage?" (*Laches* 191e10–11) and to Meno:[45]

> Even if the virtues are many and various, yet at least they all have some common form which makes them virtues. That is what ought to be kept in view by anyone who answers the question, What is virtue?[46] (*Meno* 72c6–d1)

only answer that may seem to be accounts is Meno's at *Meno* 71e1–72a2, but even in the *Meno* Meno's answer at 73d9–74a6 is clearly examples and not accounts of virtue. Laches and Euthyphro certainly describe people and actions in general terms, but the descriptions are made as the referring expressions and not intended as accounts of courage or piety. Further Hippias' beautiful maiden (*Hippias Major* 287e4) and Theaetetus' geometry (*Theaetetus* 146c8) can hardly pass for accounts of beauty or knowledge. Then why do Socrates' interlocutors confuse universals and particulars? They do not. They tell paradigmatic cases of F not because they confuse those cases with F but because those cases are heuristic devices for helping Socrates intuit the F itself.

41 See Asano 1996: 84.

42 For an argument for this claim, see Asano 1996: 90-92.

43 By contrast, Laches, Hippias, Euthyphro, and Theaetetus, although they understood this aspect of the Socratic question correctly, failed to come up with the right objects that are F and never not-F. This point is especially clear in the *Hippias Major* 289a1–d2.

44 Although the mathematicians answered the Socratic question "what is F?" they did not answer Socrates' questions about courage, beauty, piety, or virtue for a simple reason that they were interested in mathematics and not in ethics.

45 Also *Euthyphro* 6d9–11, and *Theaetetus* 148d5–7.

46 The translation of the *Meno* is Guthrie's in Hamilton and Cairns, except that I have changed the translation of *eidos* (72c7) from "character" to "form".

This aspect is captured by OMA, which infers from the premise that *many* things are (called) F in the same sense, the conclusion that there is some *one* thing, the F itself, apart from those F things.[47] As I have argued in another paper, both ACA and OMA are necessary for the introduction of Forms.[48] Thus the dialecticians who introduce Forms answer the Socratic question in both the first and the second ways.[49] This means that they can give, or at least try to give, an articulate account (definition) of F that explains why all F cases are F.

By comparison, the mathematicians who introduce a mathematical F by ACA, formally define it as an F that is never not-F,[50] and can only show what F is by ostensive definition, that is, by pointing to its images. They would say, for example, "Look at this. This is what I mean by the square."[51]

47 The principal text of this argument is *Republic* Book X, 596a6–7. For a precise analysis and interpretation of it, see Asano 1996: 84-7. The separation of the F itself from the many F things is a distinctly Platonic development of the Socratic question (cf. Aristotle, *Metaphysics* XIII. 4. 1078b30–33).

48 See Asano 1996: 88-9.

49 The self-predication of Forms is a murky problem, but Plato apparently believed that the Form of F is F (*Protagoras* 330c3–e1, *Symposium* 210e6–211a5, and *Parmenides* 132a6–8).

50 This is a circular definition because the definiens "an F that is never not-F" contains the term "F" to be defined. The mathematicians' definition of the one as something that "contains no parts in itself" (526a4) falls in this pattern, too, because "no parts (*morion . . . ouden*)" means "no plurality of parts", which is equivalent to "never not-one".

51 It would be extreme to claim that the mathematicians of Plato's time never defined a mathematical term. What matters is: did they give definitions of the *most fundamental* concepts in mathematics; and how deep or superficial were those definitions? Cf. the dispute between Hare (25–8) and C. C. W. Taylor (200–202): neither of them is completely convincing. See, however, Aristotle's testimony in the *Metaphysics* (Book I, Ch. 5, 987a19–27). His description there, although obscure, suggests that the Pythagoreans defined terms in a "superficial" (987a22) way that resembles the defect of the mathematical definitions that answer the first aspect of the Socratic question; for it suggests that they defined the term "double" as "2 that is double", which is a circular definition. For the defect of the mathematical definitions, see the preceding note 50.

So the mathematicians and the dialecticians, although they answer the Socratic question on different levels, both hypothesize certain objects and make them hypotheses. They do not use the hypotheses in the same way, however. Plato tells us:

> Mathematicians take it that (*hôs*) they know their objects (510c6) . . . and that those objects are clear to all.[52] (510d1)
> They leave the hypotheses untouched. (533c2)
>
> Dialecticians make the hypotheses not principles (*archâs*) but really hypotheses—that is, steppingstones and springboards. (511b5–6)
> They destroy the hypotheses. (533c8)

From these quotes we can infer two things: first, mathematicians make the hypotheses principles;[53] and second, they do not know that they do not know their objects,[54] whereas a dialectician would know if he/she did not know the hypotheses. The second point is a case of Socratic wisdom/ignorance,[55] and it is related to the first point, which shows the two possible and very different meanings of "*hupothesis*".[56] Lacking a piece of Socratic wisdom, mathematicians are quite content with mathematical objects, which they believe "are clear to all," and they "don't think it worthwhile to give any further account of those objects to themselves or others" (510c6–7). In their attitude to mathematical objects and Forms, they are exactly like the sight-lovers with respect to many beautiful things and Beauty itself (cf. 476b4–c7, 479e1–5): both are dreaming in that they take an image for its original. Thus the mathematicians regard and use *hupotheseis* (what are laid down) as a foundation or principles on which the mathematical sciences should be based. However, the dialecticians who look not so much to images as to their originals, regard and use *hupotheseis*[57] merely as

[52] This is my translation.

[53] This is also implied by "when the beginning is what one doesn't know" at 533c3.

[54] This is also confirmed by "these men don't possess intelligence (*noun*) with respect to the objects" (511d1–2).

[55] Socratic ignorance in the sense of the lack of Socratic wisdom.

[56] Cf. note 12 above.

[57] Although the proper objects of the dialectical knowledge (*noêsis* 511d8, *epistêmê*

assumptions that are laid down without grounds.[58]

That difference of attitude to the hypotheses between the mathematicians and the dialecticians leads them to different movements of thought. On the discussion of these movements I shall spend the next section.

3. The Upward and Downward Movements of Thought

From the hypotheses, mathematicians go to an end while dialecticians go to a beginning[59] (510b5–6). The movement of thought to an end is a downward movement while the one to a beginning is an upward movement.[60] About the mathematicians' movement of thought Plato further says:

> Beginning from the hypotheses, the mathematicians go ahead with their exposition of what remains and end consistently at the object toward which their investigation was directed. (510d1–3)

The procedure described here is quite familiar to Glaucon (510d4), and there is no doubt that it is demonstration, a chain of deductive reasonings from a set of premises to a conclusion. For example, the mathematicians prove the Pythagorean theorem that the square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides, from the hypotheses of the triangle, the right angle, the square, etc. So

533e8) are Forms, the dialecticians' hypotheses include mathematical objects as well as Forms (cf. 511d1–2, where it is implied that the dialecticians can give an account of the hypotheses which the mathematicians cannot give an account of). In other words, the dialecticians recognize mathematical objects (intermediates) and appreciate what the mathematicians are doing. This is only natural because *noêsis* could recognize the objects of *dianoia* just as pistis can recognize the objects of *eikasia*.

58 This is a point agreed upon by most interpreters. Cf. Robinson: 152, 156; Cross & Woozley: 242, 245; and Annas: 277.

59 This is a true beginning (*archê*) that is not hypothesized (510b7), in contrast to the mathematicians' beginnings (*archai*) that are merely hypotheses (511b5).

60 Cf. "above" (*anôterô*) 511a6, "goes down" (*katabainêi*) 511b8, and "going up" (*anelthontes*) 511d1.

the downward movement here is a movement of deduction from the grounds to what follows from them.

What is the upward movement, then? The *Phaedo* describes three stages of the hypothetical method: the first is to set down a hypothesis and accept as true whatever seems to accord with (*sumphônein*)[61] it (*Phaedo* 100a3–5); the second is to see if the consequences (*hormêthenta*)[62] of the hypothesis are mutually consistent or not (101d4–6); and the third is to set up another hypothesis that will give an account of the first hypothesis (101d6–7). The first stage seems what the mathematicians do: to make hypotheses and show the truth of a certain proposition from them.[63] It is not clear where the second stage can be assigned in the *Republic*'s scheme: it may be part of dialectic, or part of mathematics, or both.[64] The third stage seems what the dialecticians do,[65] and if so, it may help us understand the upward movement. According to the *Phaedo*, the third stage is not essentially different from the first: "in the same way"

61 The meaning of "*sumphônein*" is not very clear. Cf. Robinson: 126–9. Apparently "*sumphônein*" can mean either consistent with or deducible from, and the former meaning is more natural; but it seems to me that consistency can amount to deducibility if Plato assumes that the hypothesis is "relevant" in an appropriate sense to a proposition to be proved.

62 The verb "*hormaô*", which is the dictionary form of "*hormêthenta*", simply means: to proceed. Thus I mean by "consequences" what came out of the hypothesis, not necessarily logical consequences implied by it.

63 Cf. Ross: 53. It is not the case that the dialecticians do not do the first stage. But they do not stay there.

64 Part of dialectic: because the second stage is conducted when the hypothesis is challenged (*ei de tis kai ta loipa*, *Phaedo* 101d3–4), it can be considered as an examination of the hypothesis itself; and also it seems to resemble a Socratic elenchus (*Republic* 534c1–3). Cf. Ross: 57; Robinson: 170–71; and Cross & Woozley: 249–50. Part of mathematics: the second stage seems to be conducted before the challenge is met (*ouk apokrinaio heôs*, *Phaedo* 101d4); and because what is set down in the first stage is not any arbitrary hypothesis but what seems the strongest hypothesis, and a hypothesis that implies mutually inconsistent consequences cannot seem strong, the second stage can be considered as part of the selection procedures of the strongest hypothesis.

65 Cf. Ross: 53; Robinson: 171; and Cross & Woozley: 250.

(*hôsautôs*, *Phaedo* 101d7) as the first, the third stage makes another hypothesis and shows the truth of the first hypothesis from it. This process of a hypothesis turning into what is to be accounted for by another hypothesis, continues as long as a new hypothesis can be challenged (*Phaedo* 101d8). Here "another hypothesis" is not an alternative hypothesis that replaces the first one but a prior hypothesis that can account for the first one.[66] So the upward movement seems a movement of hypothesizing from a consequence to what accounts for it.

Going back to the *Republic*, we can say that the difference between the mathematicians' downward movement and the dialecticians' upward movement lies in the direction of thought: the former goes from the hypotheses to what they can account for whereas the latter goes from the hypotheses to what can account for them. Probably the best commentary on the upward movement would be the following passage in Russell's *Introduction to Mathematical Philosophy* (1):

> instead of asking what can be defined and deduced from what is assumed to begin with, we ask instead what more general ideas and principles can be found, in terms of which what was our starting-point can be defined or deduced.

Although Russell's words describe the upward movement very well, we should also be aware of a few differences between Plato and Russell.

What Russell had in mind was an ideal of axiomatization, that is, the reduction of mathematics into a few axioms, and logical rigor in deduction was utmost for him. We do not know, however, how logically rigorous Plato was in making a dialectic move upward. First, as I have argued above, a hypothesis for Plato is not strictly a proposition but primarily a thing, and yet the hypothesis of a thing can include more than one proposition, for example, that there is F and that F is so and so.[67] Second, Plato does

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66 The Phaedo, too, uses a metaphor of "above" (*anôthen*, 101d8) to describe a prior hypothesis.

67 Cf. Robinson: 132; and Cross & Woozley: 250.

not make explicit everything that is necessary for the deduction of a consequence. When he says that a consequence follows from a hypothesis, he is tacitly relying on some of "our standing assumptions," including the formal principles of logic.[68] Third, most importantly, the upward movement for Plato is more than making a higher hypothesis that accounts for a consequence. In the downward movement, whatever follows from a hypothesis is regarded as true; but in the upward movement not everything that can account for a consequence is made a hypothesis. Making a higher hypothesis involves a selection of the "best" one among the higher hypotheses (cf. *Phaedo* 101d7–8). "Best" (*beltistê*) here does not simply mean strongest from a theoretical point of view but also supremely good in a moral sense. What can you do if several hypotheses are equally strong? This is where intuition comes in.

Suppose that several hypotheses are equally strong in that they can all account for a consequence well. Clearly then it is not sufficient for a hypothesis to be able to account for a consequence, because there are several such hypotheses. A selection must be made, but according to what criterion? For Plato this criterion is one of goodness. The dialecticians must have a capacity of intuiting the goodness of things, choosing the best one among hypotheses, and finally reaching the Form of the Good as the foundation of all knowledge.[69] That is why the upward movement is essentially different from the downward movement.[70]

68 Cf. Robinson: 132–3, 168; and Cross & Woozley: 250.

69 The hypothesis the dialecticians reach at the very end of the upward movement—also called "a beginning that is free from hypotheses" (510b7) and "the beginning of the whole" (511b7)—is almost certainly the Form of the Good (517b8–c1, 532a5–b2).

70 After the description of the hypothetical method, the *Phaedo* gives us a warning not to mix up the discussions of a hypothesis and its consequences (101e1–2). Aristotle's testimony is equally clear:

> For Plato, too, was right in raising this question and asking, as he used to do, "are we on the way from or to the first principles?" There is a difference, as there is in a race-course between the course from the judges

There is another element in the upward movement. Since the dialecticians account for the hypotheses, which are things, probably by fewer hypotheses and finally by one hypothesis, the upward movement seems to involve some kind of generalization: the higher a hypothesis is, the more general it is.[71] Now Plato says:

> the person who is capable of an overview (*sunoptikos*) is dialectical while the one who isn't, is not. (537c7)

The Greek word "*sunoptikos*" comes from "*sunoraô*" which means: to see together, to see all at once.[72] When one sees things together, what is seen is their community (*koinônia*), relationship (*suggeneia*), and kinship (*oikeiotês*) with one another (531d1–3, 537c2). This means that the dialecticians who can see together see the one in the many.[73]

Now dialectic includes the downward movement as well as the upward movement. Plato says:

> When dialectic has grasped the beginning of the whole [in the upward movement], it keeps hold of what follows from this beginning and in such fashion goes back down again to an end.[74] (511b7–8)

Moreover mathematics has its own upward movement, by which the mathematicians rise up from the visible objects to the intelligible objects.[75] So both dialectic and mathe-

> to the turning-point and the way back. (*Nicomachean Ethics* I. 4. 1095a32–1095b1)

71 Moreover it seems that the higher a hypothesis is, the grander it is, because the Form of the Good, which is the highest hypothesis, is the greatest study (505a2), the principle of the whole (511b7), and makes it possible for the other things to exist as they are as well as to be known (509b6–10). Cf. Cross & Woozley: 251.

72 Cf. Liddell and Scott.

73 Cf. *Phaedrus* 265d3–4; *Parmenides* 132a2–3; *Symposium* 210a4–211d1; Adam: 137; and Robinson: 162.

74 Literally translated, the subject of the sentence is the logos itself with the power of dialectic. The part "keeps hold of what follows from" is borrowed from Grube's translation.

matics include upward and downward movements, in spite of the impression my discussion might have given to the effect that the upward movement is limited to dialectic and the downward movement to mathematics.[76] The difference between mathematics and dialectic lies in the level of their achievement; that is, the mathematicians rise up only, for example, from "the numbers in these heard accords" (531c1–2) to "numbers themselves" (525d6) while the dialecticians can see those mathematical numbers together to move up to higher and more general hypotheses.

Seeing that dialectic includes both the upward and downward movements, some interpreters took them as "the synthesis and division described in the *Phaedrus* (265–6)."[77] Plato says in the *Phaedrus* (265c8–266b1) that the two procedures of dialectic are the synthesis of several species into one genus and the division of one genus into several species. In the *Symposium* (210a4–211d1), too, the ascent to Beauty itself is a process of generalization. Thus it seems natural that the dialecticians' upward movement, in the *Republic*, to the Form of the Good is also a process of generalization, especially since Plato does not clearly separate the beautiful and the good (452e1–2; *Symposium* 201c4–5, 204d5–e4). I wonder, however, if the downward movement is a division in the *Republic*. The mathematicians' downward movement is anything but a division. Since the dialecticians, just like the mathematicians, make a hypothesis to account for a consequence, the downward movement should be the same for both of them, at least in intent:[78] to account for the consequence from the hypothesis. So we

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75 This upward movement allows some tacit generalization, because one mathematical object can account for many visible instances of it. Although mathematical objects are not generated by the one over many argument and there are many mathematical objects of one and the same kind, any one of them can account for many visible instances of it.

76 The impression is justified insofar as we consider the movements from the mathematicians' hypotheses.

77 Robinson: 163. According to him, Zeller, Heinrich Maier, and Rodier took such a view. See also Cornford 1932: 74; and Murphy: 175.

78 Although it is doubtful whether the dialecticians' downward movement is so

cannot apply the synthesis and division in the *Phaedrus* as they are to the upward and downward movements of dialectic in the *Republic*.[79]

4. The Use of Images

Next I want to discuss the other feature of mathematics, i.e., the use of images. First let us hear from Plato how mathematicians use images:

> they use visible forms besides and make their arguments about them, not thinking about them but about those others that they are like. They make the arguments for the sake of the square itself and the diagonal itself, not for the sake of the diagonal they draw, and likewise with the rest. These things themselves that they mold and draw, of which there are shadows and images in water, they now use as images, seeking to see those things themselves that one can see in no other way than with thought. (510d5–511a1)

This is a procedure familiar enough:[80] the images mathematicians use are figures drawn on paper,[81] and counters; and while speaking about them, they actually think about the intelligible objects of which the visible figures and counters are only images. But why do they use visible images, and what does it mean that they do so? These questions are connected with another question: what is the relationship between the two features of mathematics, that is, the use of images and the use of hypotheses?[82] Do mathematicians use images because they rely on hypotheses? Or do they rely on hypotheses because they use images?

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rigorous in form as the mathematicians' downward movement, that is, a demonstration.

79 Cf. Adam: 2, 174; Robinson: 163–5; and Cross & Woozley: 257.

80 As indicated by Glaucon's response, "What you say is true" (511a2). See also 526e6–527a4.

81 Although the ancient Greeks did not have paper, we usually draw figures on paper.

82 There should be some connection between the two features. For, if they were independent from each other, there would be something else besides mathematics and dialectic which would have only one of those features; but there is no hint for there being anything else. Cf. Robinson: 154–5.

There is a passage that may suggest by its order of presentation the first view that mathematicians use images because they rely on hypotheses:[83]

> a soul in investigating mathematics is compelled to use hypotheses, and does not go to a beginning because it is unable to step out above the hypotheses. *And* it uses images . . . (511a3–6. Emphasis added.)

Similarly in 510c2–511a1, Plato first explains the mathematicians' use of hypotheses and then their use of images, and the word he uses to express the latter is *pros-chrôntai* (use something beside). The prefix *pros*, which means addition, can suggest that the use of images is derivative of the use of hypotheses. But we should wonder by what logic the mathematical use of hypotheses leads to the use of images.

To the last question, an answer is given by Sinaiko: according to him (160, 162–3), the hypotheses are assumptions which should be checked against empirical data, and that is why the use of hypotheses requires the use of visible images. But mathematical hypotheses are not proved or disproved by their visible exemplifications. For example, even if the sum of the three angles of a triangle drawn on paper is found 181° by exact measurement, it does not prove that a mathematical theorem that the sum of the three angles of a triangle is 180° is false. So Sinaiko's view is wrong in assimilating mathematics to empirical sciences.[84] Another answer to the question, suggested by Robinson (155) for the sake of Jackson's view, is: since the hypotheses have not yet been proved certain, they need to be made plausible by visible images. But this suggestion is hardly plausible, either. For mathematicians are confident that the hypotheses are "clear to all" (510d1).[85]

83 This view was taken by Jackson (144–5) and Burnet (229). See also Ross (51–2) and Robinson (155).

84 Cf. Sinaiko: 159. It is a difficult question where the natural sciences should be assigned in the four-stage scheme of the Simile of the Line in the *Republic*; but most likely they belong to the sphere of opinion (*pistis*). For different views, see Nettleship: 249–51; and Raven: 158–9.

85 Indeed, the two textual considerations adduced for the first view in the previous paragraph are very weak. The mere order of presentation does not say anything

On the other hand, there is also a passage that suggests the second view that mathematicians rely on hypotheses because they use images:[86]

> a soul, using (*chrômenê*) as images the things that were previously imitated, is compelled to investigate on the basis of hypotheses . . . (510b4–5)

In this passage, where Plato first introduces the two features of mathematics, the participle *chrômenê* is naturally taken as giving a reason why a soul is compelled to investigate on the basis of hypotheses.[87] Again at 511a4, mathematicians are said to be "compelled" to use hypotheses. Compelled by what? The reason is given: because they are "unable to step out above the hypotheses"(511a5–6).[88] Then why are they not able to do so? The quote (511a5–6) is immediately followed by the mention of the use of images (511a6–7): this can give an impression that they are not able to step out above the hypotheses because they use images.[89]

Some textual considerations for the second view that mathematicians rely on hypotheses because they use images, may seem rather weak, but others confirm it. The participle *chrômenê* may simply express a concurrent action, or perhaps a means for the use of hypotheses. That a negative expression is followed by a positive expression does not necessarily mean that the latter gives the reason for the former, because the latter can be a consequence of the former. There is, however, one point we should insist on:

definite: what is presented later can be more fundamental than what is presented first. The use of the prefix *pros* may mean a mere addition without implying any relation between what is added and what it is added to.

86 Most interpreters take this view. Cf. Ross: 52; Murphy: 172 and 177; Robinson: 156; Cross & Woozley: 244–5; N. White 1976: 96, 111–12, note 43; and Annas: 278.

87 Cf. Ross: 52; and Robinson: 154–5.

88 Precisely speaking, this is given as a reason why mathematicians do not go to a beginning; but in the context, using hypotheses and not going to a beginning mean the same thing.

89 Although in the grammatical structure the conjunctive particle *de* (but, and) at 511a6 connects the mention of the use of images (511a6–8) with the mention of the use of hypotheses (511a3–4) while the intervening 511a5–6 is an addition in parentheses.

mathematicians are *compelled* to use hypotheses. This clearly means that the use of hypotheses comes from what compels mathematicians to do so; and the only thing that we can think of as compelling them to do so is the other feature of mathematics, i.e., the use of images.

Perhaps we can better understand this from the nature of the two features. The use of images is not only a feature distinguishing mathematics from dialectic, but more importantly it is what distinguishes mathematics from opinion. One should not forget this simply because it is obvious. It is, as a matter of fact, what has elevated mathematics above opinion, and is related to mathematics' upward movement, that is, from the visible figures and counters to the intelligible objects. The use of hypotheses, on the other hand, is related to mathematics' downward movement, that is, from the hypothesized objects to whatever follows from them. So the use of images must come before the use of hypotheses just as the upward movement should naturally precede the downward movement. For unless mathematicians go up to the intelligible objects first, they cannot go down from them at all. Thus the use of images is more fundamental, and it is something that determines what mathematicians can do and cannot do afterwards.

Then how does the use of images compel mathematicians to rely on hypotheses and to be unable to step out above them? It has to do with the other aspect of the use of images. Images both lift up and keep down people. We have already seen that images lift up mathematicians to the intelligible objects; but the other effect of images is that the mathematicians' understanding of intelligible objects is dependent on images and cannot get rid of a constraint imposed by them. The constraint I mean is the spatiality of images. Since the visible figures and counters are spatial, mathematicians imagine their intelligible objects to be spatial, too. For example, just as a triangle on paper is triangular, having a certain shape and size, the mathematicians' triangle is also triangular, having a ceratin shape and size. The only difference is that the former is visible whereas the latter is not visible with the eye, yet imaginable with the mind's eye (cf. Crombie, Vol. 1: 130).

The last point is directly related to the nature of mathematical objects and mathematics as a study about those objects and their relations. As I have argued above,[90] mathematicians define a mathematical object F as what is F and never not-F. Thus there is no doubt that, for example, a mathematical triangle is a triangle. Moreover, it is not just a triangle but an idealized triangle, which, being abstracted from matter, never fails to be a triangle in any way. That is why mathematical objects cannot be seen with the eye but can be seen with the mind's eye. This way of conceiving mathematical objects is not necessarily wrong: at least the ancient Greeks took mathematics to be a science about space and spatial entities (cf. Cross & Woozley: 239–40), including units, numbers, two-dimensional and three-dimensional figures.[91]

We can now answer the questions we have set at the beginning of this section. Mathematics uses images, because they are fairly close approximations to the intelligible objects—close enough to let mathematicians intuitively grasp those objects. For they are spatial images of spatial objects. So using the images is a good method for the study of those objects. What this means is that mathematicians' intelligible objects are of the same nature as the visible figures and counters in an important way: the former is just as spatial as the latter.[92] Since mathematicians understand their intelligible objects on the basis of images, they believe that those objects are "clear to all" (510d1) who see the images.

90 In the later part of Section 2.

91 Arithmetic and algebra are not exceptions, for the ancient Greeks understood them in a geometrical manner, representing units and numbers as dots and their collections. Cf. Ross: 49–50. Interestingly, in a later work *Timaeus* (48e2–49a6, 50c7–d2, 51e6–b5) Plato makes space one of the three natures of the universe, along with Forms and becoming, although the relationship between space of the *Timaeus* and mathematical objects is totally obscure (for one thing, space is formless while mathematical objects are not so).

92 To borrow Crombie's words, the objects of mathematics are non-empirical, yet semi-empirical. Cf. Crombie, Vol. 1: 110, 118, 124–6.

Further, mathematical objects, being spatial, are individuated by space; that is, there can be the same things, for example, the same units, here and there.[93] Seeing those units, dialecticians would wonder and ask what it is that makes them all units. But to ask that question is beyond the capacity of mathematicians, because they believe that mathematical objects are "clear to all", and "don't think it worthwhile to give any further account of them to themselves or others" (510c6–7). That is why they cannot step out above the hypotheses but have to rely on them.[94]

By contrast, dialecticians do not use images at all: what they deal with are Forms only (510b7–9, 511c1–2). Since Forms are each unique (479a4, 507b6, and 596a6)[95] and cannot be individuated by space or time, they are neither spatial nor temporal (cf. *Symposium* 210e6–211a4). So they are radically different from their images in space and time. Thus the images are more misleading than instructive for the cognition of Forms, and that is why dialecticians do not use images; instead, the only thing they rely on is the power of discourse (511b4).

5. Mathematics as the Prelude to Dialectic

The last question I want to discuss is the most difficult one in this paper. Mathematics is singled out as the prelude to dialectic, which ultimately aims at the Form of the Good (532a5–b2). The question is: how does mathematics have anything to do with ethics? It should be noted at the beginning that since the textual evidence on this matter is scanty, the following discussions cannot avoid being quite speculative.

As we have already seen, mathematics has the power to lift up the mind from the

93 For the plurality of mathematical objects, see Asano 1996: 88-92.

94 Theoretically speaking, there is nothing that prevents one and the same person from becoming a mathematician and a dialectician at once (cf. 531e2–3). That is why mathematics can be a good prelude to dialectic; but *insofar as* people are mathematicians, they do not ask dialectical questions.

95 For the uniqueness of Forms, see Asano 1996: 84-92.

visible to the intelligible objects. Thus there is at least one reason why mathematics is singled out as the prelude to dialectic: it is a training of logical thinking.[96] That is the whole reason, according to one interpretation (Cross & Woozley: 254–6). There is no direct link between mathematics and ethics; that is, there is nothing particularly mathematical about ethics, and nothing particularly moral about mathematics. However, mathematics is just useful as a preliminary training for dialectic in that it turns one's mind from the world of becoming to the world of being. But that does not seem to be a reason sufficient for explaining Plato's insistence on mathematics in his educational program for the future rulers (521c1–531e1). For example, does he not find any significance in the other feature of mathematics, i.e., the use of hypotheses?

According to another view, mathematics, because of its hypothetical method, is a model for any serious science.[97] The hypothetical method consists in constructions of premises, deductions of conclusions, a clear distinction between conclusions and premises, and an open confession of premises.[98] Those are the characteristics of

[96] What Nettleship calls "mental gymnastic" (270). The necessity of a preliminary training for philosophy is voiced in the *Parmenides*, too:

> That is because you are undertaking to define "beautiful," "just," "good," and other particular forms, too soon, before you have had a preliminary training. I noticed that the other day when I heard you talking here with Aristoteles. Believe me, there is something noble and inspired in your passion for argument, but you must make an effort and submit yourself, while you are still young, to a severer training in what the world calls idle talk and condemns as useless. Otherwise, the truth will escape you. (135c8–d6)

Although the preliminary training meant in the *Parmenides* is different from mathematics, it is possible that the *Republic* conceives mathematics as the preliminary training in a similar manner.

[97] Robinson: 153, 177–8; and Annas: 272–4, 287–90.

[98] Robinson: 177. In Annas' version of this view, mathematics serves as a model because of its hypothetico-deductive procedure, which achieves an understanding of a rationally organized system where it is clear what is basic, what derived, and how each result depends on what has gone before (Annas: 289–90).

mathematics which the future rulers should learn so that they could pursue ethics in a similar methodical manner. Thus, according to this view, there is nothing particularly moral about mathematics; but dialectic is mathematical in method and when it reaches the Form of the Good, it will be *ordine geometrico demonstrata* (demonstrated in geometrical order) (Annas: 288, 290).

It may seem odd at first sight that mathematics is a model of dialectic, when Plato contrasts them by saying how mathematics is inferior to dialectic. Certainly the hypothetical method as practiced by mathematicians could not be a model of dialectic, because they practiced it defectively in the two ways for which Plato criticized them: the use of images, and the consequent, dogmatic acceptance of hypotheses. According to the above view, however, if purified of those defects, the hypothetical method can become the method of dialectic (Robinson: 177–8). So dialecticians will use the hypothetical method without using images or leaving the hypotheses unquestioned.

This view is sensible: it duly recognizes both how dialectic is similar to mathematics and how it is not so. We can hardly doubt that both mathematics and dialectic use the hypothetical method or that the one does not have the two defects for which the other is criticized. Nevertheless, the view does not seem to explain the intermediate status of mathematics well. Certainly dialectic uses the hypothetical method, but the hypothetical method is not particularly mathematical. Then why should the future rulers study mathematics? It is not because mathematics is valuable in itself; but simply because it "was the only branch of knowledge familiar to Plato which had advanced in" using the hypothetical method (Annas: 289). So what really matters is the hypothetical method, according to this view, and if a curriculum of the hypothetical method was worked out, mathematics could be dispensed with.

But what other disciplines could Plato have thought of as the preliminary study to dialectic? As he says, "all the other arts are directed to human opinions and desires, or to generation and composition, or to the care of what is grown or put together"

(533b3–6). This suggests that mathematics is chosen as the preliminary study not because its hypothetical method is unique but because its objects are unique and uniquely exempt from change. If so, mathematics cannot be replaced by other curricula of the hypothetical method.

According to still another view, there is something good about the objects of mathematics, and the study of them is the first step towards understanding the Good.[99] In a weaker version of the view, mathematical objects are good because they are the elementary principles of being and "everything in the world is ultimately a manifestation of the divine intelligence" (Nettleship: 270). This version, however, does not distinguish mathematics from the other sciences: mathematics is only the elementary part of the sciences; and that is why mathematics comes first. But, as we have seen just above, Plato does not conceive any other science but mathematics as the prelude to dialectic.[100]

In a stronger version of the view, mathematical objects are good in a special way.[101] They are good because they are spatial images of the Forms, which are good things.[102] Although the visible objects are also images of Forms, they are material images, and hence they are muddled, i.e., show conflicting appearances. Unlike the material images, the mathematical objects are pure and clear. Moreover, every Form has mathematical images. For example, 4 may be an image of justice.[103] Hence to study

99 Nettleship: 269–70; and Crombie, Vol. 1: 124–6 and 132–3.

100 Neither does he conceive the natural sciences as part of dialectic.

101 Crombie, Vol. 1: 124–6. See also Gosling 1973: 101.

102 Because it is the Form of the Good that makes the other Forms be what they are (509b7–8), they would be good things in spite of Plato's occasional mention to negative forms such as ugliness (475e9), injustice and evil (476a4). See Santas for the relation between the Form of the Good and the other Forms.

103 The thought behind this example of the Pythagorean origin may be that 4 is the first and typical square and that a square is an expression of reciprocity (Crombie, Vol. 1: 125). Although it may sound like a fancy, it is possible that Plato may have had such a mathematical understanding of Forms. Plato was clearly influenced by the Pythagoreans (cf. Aristotle, *Metaphysics* I. 6. 987a29–31).

mathematics is to “lay hold of something of what is” (533b7). I find this version attractive. For it seems to explain well the unique position of mathematics in Plato’s educational program for the future rulers.

What then is so good about the mathematical objects? I think that we tend to view mathematics as something unrelated to ethics because we view it only as something true. But for Plato mathematics is not only true but equally beautiful. He writes of the beauty of the sciences in the *Symposium* (210c6–7):

> And next, his attention should be diverted from institutions to the sciences, so that he [the initiate] may know the beauty of every kind of knowledge (*epistêmôn kallos*).[104]

This is a stage in the ascent of eros between the one in which the initiate loves the beauties of the soul, laws and institutions, and the one in which he/she loves the Beauty itself. This stage might be fairly regarded as corresponding to the mathematical education in the *Republic* (cf. Cornford 1950A: 126–7). Thus the major reason why the future rulers should study mathematics is that it is beautiful, or we might say elegant. Since Plato does not clearly separate the beautiful and the good, mathematics has a moral significance for him.

Lastly there is a view, which seems to go further in this direction (Gosling 1973: 100–07, 117–19). According to Gosling, there is a distinction among the five branches of mathematics, i.e., arithmetic (522c5–526c8), plane geometry (526c8–527c11), solid geometry (528a9–e2), astronomy (528e3–530c5), and harmonics (530c5–531c8), in Plato’s educational program. Arithmetic and geometry are pure mathematics which is non-evaluative: they are concerned with the odd, the even, the figures, etc., but not with the good and the bad. Pure mathematics is necessary but not sufficient as the preliminary study to dialectic. Astronomy and harmonics on the other hand are “honorable” mathematics which involves more directly the notion of the good. For

[104] The sciences are also called *mathêmata* (211c6). The translation of the *Symposium* is Joyce’s in Hamilton and Cairns.

they "are concerned with good and bad proportions and arrangements" (Gosling 1973: 104). This means that they can tell us, for example, at what speed a star should move (cf. 529d1–4), and what proportion can make a harmonic sound (cf. 531c3). Thus according to Gosling, at least astronomy and harmonics are already moral in that they study not simply mathematical structures of things but proper, i.e., good structures of things in mathematical terms (Gosling 1973: 101, 103). That may seem to make it easier to understand why mathematics can be a good preparation for the study of the Form of the Good.

Gosling assimilates astronomy and harmonics to practical disciplines such as medicine and navigation. Plato, however, distinguishes mathematics and vulgar arts (533b3–6). Then what is the point of distinguishing astronomy and harmonics from the vulgar arts? It is, according to Gosling, to distinguish the proper sciences from the vulgar arts (1973: 106): first the vulgar arts are conditioned by immediate human needs while the proper sciences are detached from them, and second the vulgar arts concentrate on observable properties of things while the proper sciences develop a precise mathematical account (cf. 529a9–530c1, 530e5–531c4). The role of pure mathematics is to raise the vulgar arts to the proper sciences, specifically to help develop a theoretical interest, and to give precision to disciplines (Gosling 1973: 117–18).

Gosling's view is a bit complicated as it divides mathematics into two sorts: pure mathematics (arithmetic and geometry) and honorable mathematics (astronomy and harmonics). Pure mathematics is only a tool for honorable mathematics, and preference is given to the latter (Gosling 1973: 118). But, even though that might be an interesting development of Plato's thought in his later years, there is simply no textual support for it in the *Republic*. First the *Republic* does not distinguish the two sorts of mathematics. Second, arithmetic and geometry are far more prominent than astronomy and harmonics: in the Simile of the Line there is no mention of astronomy or harmonics; and in *Republic* Book VII astronomy as the study of the motion of what has depth

(528e1) is conceived as a natural extension of solid geometry, and Plato says that astronomy and harmonics should be studied in the same way in which geometry is studied, i.e., by using as images what is visible or audible, and using hypotheses (529c7–530c1, 530e7–531c4).[105]

So Plato, assimilating astronomy and harmonics to pure mathematics, seems to be turning them into *a priori* sciences rather than empirical ones. This suggests that the truths of astronomy and harmonics as Plato understood them, are necessitated by their logic as much as those of arithmetic and geometry. Then are astronomy and harmonics really concerned with the good and the bad as Gosling claims? What Plato says is:

> 1. [In astronomy] the true movements in which the really fast and the really slow—in true number and in all the true figures—are moved . . . must be grasped by argument and thought, not sight. (529d1–5)
> 2. [One cannot] grasp the truth about equals, doubles, or any other proportion (*summetria*) in the visible things. (529e5–530a1)
> 3. As for the proportion (*summetria*) of night to day, of these to a month, of a month to a year, and of the rest of the stars to these and to one another . . . [the stars are not] always the same [in movement] but deviate . . . for they are connected with the body and are visible. (530a7–b3)
> 4. [People who practise harmonics in a wrong way] seek the numbers in these heard accords (*sumphônia*) and don't rise to the problems, to the consideration of which numbers are concordant (*sumphônos*) and which not, and why in each case. (531c1–4)

The first three quotes simply state that the movements and proportions astronomy is concerned with are intelligible ones and not visible ones. Further the second quote actually refers to a geometer, and *summetria* is a term in geometry.[106] The only quote that might suggest the value-ladenness of the objects of harmonics is the last one,

105 "The use of problems" (530b6) refers to geometry's use of hypotheses to solve the problems. Cf. Burnet (222): "Simplicius [in *de caelo*] . . . tells us that Plato, who held that the movements of the heavenly bodies must be regular, 'propounded it as a problem' to the mathematicians of the Academy to find on what hypotheses (*tinôn hupotethentôn*) their apparent irregularity could be explained so as to 'save the appearances'."

106 *Summetria* is a noun form of *summetros*, which means: commensurate with. Cf. *Theaetetus* 147d5–6, 148b1.

especially the term *sumphônos*. Yet, here again, what matters for Plato seems not so much agreeable sounds as an agreeing of numbers in some mathematical character; and why such and such numbers are concordant would be explained from, among others, the hypothesis of concordance. So, after all, astronomy and harmonics are not more value-laden than arithmetic and geometry. All of them are beautiful and good, but none is more so than others.

6. Conclusion

In this paper I have discussed the four major problems concerning the differences and relations between mathematics and dialectic. First, I have argued, the hypotheses of mathematics are mathematical objects. The hypotheses at the same time involve the definitions of those objects as understood by the mathematicians, and existential and other propositions concerning those objects. What distinguishes mathematicians and dialecticians is their respective attitude to the hypotheses: the former accept them as obvious principles while the latter try to account for them further. Second, the upward and downward movements of thought are generally distinguished by the direction in explanation. The upward movement goes from something to what can account for it, and the downward movement from something to what can be accounted for by it. However, those movements are found in both mathematics and dialectic. Third, mathematicians use images as an aid because the objects of their study are spatial entities, to which the spatial images can be a fairly close approximation. And that expedient method is what compels mathematicians to rely on hypotheses. Fourth, mathematical studies are the only appropriate prelude to dialectic because they are *a priori* sciences and their objects, being pure images of good Forms, are uniquely beautiful and good.

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Bibliography

Adam, James. *The Republic of Plato*. Vol. 2. 2d ed. Cambridge: Cambridge University Press, 1963.

Allen, R. E., ed. *Studies in Plato's Metaphysics*. London: Routledge and Kegan Paul, 1965.

Annas, Julia. *An Introduction to Plato's Republic*. Oxford: Clarendon Press, 1981.

Anton, J. P., and Preus, A., eds. *Essays in Ancient Greek Philosophy*, Vol. II. Albany, NY: State University of New York, 1983.

Archer-Hind, R. D. *The Phaedo of Plato*. 2d ed. 1894. Reprint, New York: Arno Press, 1973.

Asano, Kozi. [1993]. "Degrees of Reality in Plato: Part I." *Aichi* (*Philosophy*) Vol. 10: 131–118.（神戸大学哲学懇話会『愛知』第10号，131–118頁）

———. [1994]. "Degrees of Reality in Plato: Part II." *The Hannan Ronshu* (*Journal of Hannan University*) *Humanities & Natural Science* Vol. 30, No. 2 (Sep.): 17–34.（阪南大学学会『阪南論集　人文・自然科学編』第30巻第２号，1994年９月，17–34頁）

———. [1996]. "Two Arguments for Forms in Plato: Conflicting Appearances and One over Many." *The Hannan Ronshu* (*Journal of Hannan University*) *Humanities & Natural Science* Vol. 32, No. 1 (June): 79–96. （阪南大学学会『阪南論集　人文・自然科学編』第32巻第１号，1996年６月，79–96頁）

———. [1997A]. "The Simile of the Line in Plato's *Republic* VI." *Sapientia* (*The Eichi Uni-versity Review*) No. 31: 207–34.（英知大学『サピエンチア』第31号，207–34頁）

———. [1997B]. "A Study of Plato's Metaphysics in the *Republic*." Ph.D. diss., University of Texas, Austin.

Bambrough, Renford, ed. *New Essays on Plato and Aristotle*. London: Routledge & Kegan Paul, 1965.

Barnes, Jonathan, ed. *The Complete Works of Aristotle*. Vol. 2. Princeton: Princeton University Press, 1985.

Bloom, Allan, trans. *The Republic of Plato*. 2d ed. New York: Basic Books, 1991.

Burnet, J. *Greek Philosophy*. London: Macmillan, 1914.

Cooper, Neil. "The Importance of *dianoia* in Plato's Theory of Forms." *Classical Quarterly* 16 (1966): 65–9.

Cornford, F. M. [1932]. "Mathematics and Dialectic in the *Republic* VI–VII." *Mind*

41: 37–52, 173–90. Reprinted in Allen 1965: 61–95. (Cited in the latter pagination.)

———. [1950A]. "The Doctrine of Eros in Plato's *Symposium*." In his 1950B: 68–80. Re-printed in Vlastos: 119–31. (Cited in the latter pagination.)

———. [1950B]. *The Unwritten Philosophy and Other Essays*, edited by W. K. C. Guthrie. Cambridge: Cambridge University Press.

Crombie, I. M. *An Examination of Plato's Doctrines*. 2 Vols. London: Routledge & Kegan Paul, 1962.

Cross, R. C., and Woozley, A. D. *Plato's Republic: A Philosophical Commentary*. London: Macmillan, 1964.

Gallop, David, trans. *Plato: Phaedo*. Oxford: Clarendon Press, 1975.

Gosling, J. C. B. [1960]. "*Republic* V: *ta polla kala* etc." *Phronesis* 5: 116–28.

———. [1968]. "*Doxa* and *Dunamis* in Plato's *Republic*." *Phronesis* 13: 119–30.

———. [1973]. *Plato*. London: Routledge and Kegan Paul.

Grube, G. M. A., trans. *Plato's Republic*. Indianapolis: Hackett Publishing, 1974.

Guthrie, W. K. C. *A History of Greek Philosophy IV Plato, the Man and his Dialogues: Earlier Period*. Cambridge: Cambridge University Press, 1975.

Hamilton, E., and Cairns, H., eds. *The Collected Dialogues of Plato*. Princeton: Princeton University Press, 1963.

Hardie, W. F. R. *A Study in Plato*. Oxford: Clarendon Press, 1936.

Hare, R. M. "Plato and the Mathematicians." In Bambrough: 21–38.

Jackson, Henry. "On Plato's *Republic* VI 509d sqq." *Journal of Philology* 10 (1882): 132–50.

Kraut, Richard, ed. *The Cambridge Companion to Plato*. Cambridge: Cambridge University Press, 1992.

Liddell, H. G. and Scott, R. *A Greek-English Lexicon*. 9th ed. Oxford: Clarendon Press, 1968.

Murphy, N. R. *The Interpretation of Plato's Republic*. Oxford: Clarendon Press, 1951.

Nehamas, Alexander. "Confusing Universals and Particulars in Plato's Early Dialogues." *Review of Metaphysics* 29 (1975): 287–306.

Nettleship, R. L. *Lectures on the Republic of Plato*. 2d ed. London: Macmillan, 1901.

Raven, J. E. *Plato's Thought in the Making: A Study of the Development of his Metaphysics*. Cambridge: Cambridge University Press, 1965.

Reeve, C. D. C. *Philosopher-Kings: The Argument of Plato's Republic*. Princeton: Princeton University Press, 1988.

Robinson, Richard. *Plato's Earlier Dialectic*. 2d ed. Oxford: Clarendon Press, 1953.

Ross, David. *Plato's Theory of Ideas*. Oxford: Clarendon Press, 1951. Reprint, Westport, CT: Greenwood Press, 1976.

Russell, Bertrand. *Introduction to Mathematical Philosophy*. 2d ed. Reprint, New York: Dover Publications, 1993.

Santas, Gerasimos. "The Form of the Good in Plato's *Republic*." In Anton and Preus: 232–63.

Sinaiko, H. L. *Love, Knowledge, and Discourse in Plato: Dialogue and Dialectic in Phaedrus, Republic, and Parmenides*. Chicago: University of Chicago Press, 1965.

Taylor, A. E. "A Note on Plato's *Republic* VI 510c2–5." *Mind* N. S. 43 (1934): 81–4.

Taylor, C. C. W. "Plato and the Mathematicians: an Examination of Professor Hare's Views." *Philosophical Quarterly* 17 (1967): 193–203.

Vlastos, Gregory, ed. *Plato: A Collection of Critical Essays. Vol. II, Ethics, Politics, and Philosophy of Art and Religion*. Garden City, NY: Anchor Books, 1971. Reprint, Notre Dame, Indiana: University of Notre Dame Press, 1978.

White, Nicholas P. [1976]. *Plato on Knowledge and Reality*. Indianapolis: Hackett Publishing.

———. [1979]. *A Companion to Plato's Republic*. Indianapolis: Hackett Publishing.

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