

## The Simile of the Line in Plato's *Republic* VI<sup>1</sup>

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### 1. Introduction

In the *Republic* Book V (476a9-480a13), Plato distinguished the philosophers and the sight-lovers in terms of the difference of the objects of their cognition and love; that is, the philosophers are those who recognize and love what is,<sup>2</sup> while the sight-lovers are those who recognize and love what is and is not. What is is the knowable (*gnôston*), i.e., the proper object of knowledge (478b3), and what is and is not is the opinable (*doxaston*), i.e., the proper object of opinion (478e1-4). Since the philosophers' cognition is related to what is, their cognition is called knowledge (479e7-9); whereas the sight-lovers' cognition, being related to what is and is not, is called opinion (478e1-479e6). The next question one would be inclined to ask is how those two kinds of objects of cognition, what is and what is and is not, are connected with one another.<sup>3</sup>

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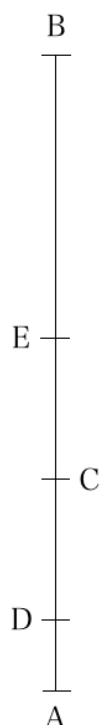
<sup>1</sup> This paper is part of my Ph. D. dissertation to be submitted to the University of Texas at Austin. Part of the paper has been published in Japanese under the title "The Objects of Mathematics in Plato's *Republic*."

<sup>2</sup> One should not forget that the philosophers recognize what is and is not, too (476c9-d4), although it is not clear if they also love what is and is not. The description of the philosophers in Book VII suggests that they do not; and this view may be reinforced by the hydraulic theory of psychic energy, according to which as the total amount of psychic energy is limited, the more psychic energy flows into one direction, the less flows into the other directions (485d6-9). On the other hand, the Book V characterization of love of X as loving every bit of what is described under X (474c8-475b7), can suggest that the philosophers who love the Beauty love whatever is and is not beautiful insofar as it participates in the Beauty.

<sup>3</sup> It is fairly clear that there is some kind of connection between what is and what is and is not; for what is and is not, as something intermediate, shares being with what is, and not-being with what is not (478e1-2). Although there are two hints as to the nature of the connection in Book V, they are not so helpful by themselves. One is the characterization of the sight-lovers as dreaming in 476c2-d7, where it is implied that as something is to what is like it, so what is is to what is and is not. But this amounts only

This is the major problem Plato tries to explicate in the Simile of the Line toward the end of Book VI (509d1-511e5).<sup>4</sup>

In the rest of the introduction, I shall outline the Simile of the Line; then I shall discuss the four major interpretative problems of the Simile in the following sections.



The Simile of the Line takes over the two kinds of objects of cognition, the visible (*horaton*) and the intelligible (*noêton*), from the preceding Simile of the Sun (506d6-509c11).<sup>5</sup> The Simile of the Sun took it for granted that the visible and the intelligible are of two distinct classes (507b2-11), and explained the status of the Good within the intelligible world by analogy to that of the sun within the visible world (508b13-c2). The Simile of the Sun, however, has left unspecified the relation between the visible and the intelligible much the same way as the discussion in Book V, mostly concerned with distinguishing the knowable and the opinable, has left unspecified their relationship.

Now imagine a line AB divided into two unequal segments, AC and CB, and let the shorter segment AC represent the visible and the longer segment CB the intelligible.<sup>6</sup> Then divide each segment in the same ratio to the statement that there is a resemblance between what is and what is and is not; and one cannot know the nature of the resemblance between them unless one is previously informed of the specific way in which something is connected with what is like it. The second hint is the description of the relation of what is and is not to what is as participation (476d1-3). Even though “participation” suggests some kind of priority of what is over what is and is not, it is no more informative than “resemblance” as to the precise nature of the connection between them.

<sup>4</sup> This is the major point of the Simile of the Line: to make a connection of what is and what is and is not. There are other problems, too; among others, Plato is also quite concerned with making some kind of distinction within what is. There is, however, a dissenting interpretation by Ferguson. I shall discuss his interpretation in the next section.

<sup>5</sup> And less directly from the distinction between what is, i.e., the knowable, and what is and is not, i.e., the opinable, in Book V, too.

as the first division: AC into AD and DC, and CB into CE and EB (509d6-8). Within the visible segment AC, the shorter segment AD consists of images, by which Plato means “first shadows, then appearances produced in water and in all close-grained, smooth, bright things, and everything of that sort” (509e1-510a3).<sup>7</sup> The longer segment DC consists of their originals, i.e., “the animals around us, and everything that grows, and the whole class of artifacts” (510a5-6).<sup>8</sup> The major point of the Simile of the Line is that  $AC : CB = AD : DC$ . This means that with respect to truth or lack of it, as images are to their originals, so the visible is to the intelligible (510a8-b1).<sup>9</sup> Then

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<sup>6</sup> There is an implausible manuscript reading (*an, isa*) and its equally implausible emendations (*isa; an isa*; etc.), according to which the line is bisected into equal parts. Cf. N. R. Murphy, “The ‘Simile of Light’ in Plato’s *Republic*,” 99, note 1, and *The Interpretation of Plato’s Republic*, 158-9. The bisections, however, would produce the four equal segments, and deprive the Simile of the Line of any sense of mathematical proportion. For criticisms of this bisection theory, see also R. L. Nettleship, *Lectures on the Republic of Plato*, 239, editor[G. R. Benson]’s note; J. Adam, *The Republic of Plato* Vol. II, 64; D. Ross, *Plato’s Theory of Ideas*, 45, note 2; and W. K. C. Guthrie, *A History of Greek Philosophy IV Plato, the Man and his Dialogues: Earlier Period*, 508, note 3.

<sup>7</sup> I shall use Bloom’s translation of the *Republic*, with minor changes necessary to fit the context of my paper.

<sup>8</sup> This order, in which images and their originals are presented to the reader, is significant; for if originals were presented first and their images later, then the images would hardly be interesting to the reader. Cf. the definition of dreaming in Book V (476c5-7). The movement of thought from originals to their images would not bring out anything new. But the movement of thought from images to their originals is accompanied by new insight, which can be properly described as awakening, i.e., coming to know better what one was seeing before.

<sup>9</sup> The structure of Plato’s sentence is: as the opinable is to the intelligible, so the images are to their originals (510a9-10). But it would be perverse to take Plato as instructing us to understand the relation between the images and their originals from the relation between the opinable and the intelligible, because the images and their originals are much more familiar to us than the opinable and the intelligible. To put it differently, among the four terms of the proportion, images, originals, the opinable, and the intelligible, the intelligible is the only unknown, or hard-to-know, term. So I think that the thought underneath the sentence is something like this: “Do you wonder about the relation between the opinable and the intelligible? It is the same as this

Plato goes on to note another major point of the Simile of the Line that  $AD : DC = CE : EB$ . This means that with respect to truth or lack of it, as images are to their originals, so the objects in CE are to the objects in EB. Plato calls the cognition of the objects in EB *noêsis*, the cognition of those in CE *dianoia*, the cognition of those in DC *pistis*, and the cognition of those in AD *eikasia* (511d6-e2).<sup>10</sup> The Simile of the Line ( $AC : CB = AD : DC = CE : EB$ ) expresses the relative truth of the objects in those segments, and the relative clarity of their cognitions as well (511e2-4).

There are the four major interpretative problems of the Simile. The first is the significance of the visible (*horaton*): exactly what is meant by the visible? The second is the significance of *eikasia*: what kind of cognitive state is it? The third is the equality of the two middle segments, DC and CE, of the line. Is it merely an unintended consequence of the Simile, or does it have any meaning? The fourth and biggest problem is the objects of *dianoia*. Are they Forms, or “intermediates”<sup>11</sup>? I shall discuss these problems in the rest of the paper.<sup>12</sup> The overall meaning of the Simile of the Line will, I hope, become clear through my discussion.

## 2. The Visible (*horaton*)

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relation between the images and their originals.”

<sup>10</sup> It is hard to translate these Greek words for the four types of cognition corresponding to the four segments EB, CE, DC, and AD of the line. The cognition of the intelligible, the objects in CB, is knowledge (508d6), and the cognition of the visible, the objects in AC, is sight (508d1-2). But it is not clear why AC is longer than DC, i.e., how sight is clearer than *pistis*, nor why CB is longer than EB, i.e., how knowledge is clearer than *noêsis*. Perhaps it is not intended; for the extent to which the visible participates in truth must be at most the same as that to which the objects in DC participate in truth, and the extent to which the intelligible participates in truth must be at most the same as that to which the objects in EB participate in truth.

<sup>11</sup> The objects Aristotle calls “intermediate” or “mathematicals”. Cf. Aristotle, *Metaphysics*, I 6, 9; and XIII 6-7.

<sup>12</sup> There is one very big issue that is not discussed in the present paper because it is too big a topic to include here: the issue is the mathematician’s hypothetical method and the philosopher’s dialectical method. I will write another paper on that issue.

I have written in the above that the major point of the Simile of the Line is to connect what is (the intelligible) and what is and is not (the visible). There is, however, a serious dissenting interpretation offered by Ferguson<sup>13</sup> as to the significance of the visible (*horaton*) in the Simile of the Line. According to Ferguson, the visible in the Simile of the Line does not stand for the sensible in general, but it is simply visual colors and shapes.<sup>14</sup> If so, the Simile of the Line cannot be about the relation between the *sensible* (what is and is not) and the intelligible.<sup>15</sup> In this section, I shall discuss this dissenting interpretation, and then defend my reading of the visible as representative of what is and is not, i.e., the sensible particulars.

First of all, the Simile of the Line is a continuation of the Simile of the Sun (509c5-10).<sup>16</sup> So let us look at the way in which the visible was introduced and used in the Simile of the Sun. There sight was picked up from among the senses as requiring a third kind of thing, i.e., light, in the absence of which sight cannot see and a color cannot be seen (507d8-e5). Thus sight is contrasted with the other senses, e.g., hearing, which

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<sup>13</sup> A. S. Ferguson, "Plato's Simile of Light: Part I. The Similes of the Sun and the Line." The same view is also taken by J. E. Raven, "Sun, Divided Line, and Cave," 22-32, and *Plato's Thought in the Making: A Study of the Development of his Metaphysics*, 140-51.

<sup>14</sup> Ferguson, "Plato's Simile of Light: Part I," 133, 136-7, and "Plato's Simile of Light Again," 193.

<sup>15</sup> This does not trouble Ferguson, because he thinks that the relation between the knowable and the opinable has been already established in Book V ("Plato's Simile of Light: Part I," 143, and "Plato's Simile of Light Again," 197). But I disagree. Certainly it is established in Book V that what is is more real than what is and is not (and that knowledge is clearer than opinion). But that is the difference, and not the connection, between what is and what is and is not. Cf. note 3 above concerning the not-so-helpful hints given in Book V about the nature of the connection.

<sup>16</sup> As rightly observed by Ferguson ("Plato's Simile of Light: Part I," 136), Raven ("Sun, Divided Line, and Cave," 31, and *Plato's Thought*, 142, 146-7, 150) and R. C. Cross & A. D. Woozley (*Plato's Republic: A Philosophical Commentary*, 203 and 209).

do not need any third kind of thing to perceive their objects, e.g., sounds (507c10-d7).<sup>17</sup> So the visible, i.e., the objects of sight, there cannot stand for the objects of the other senses, and hence it cannot stand for the sensible in general nor the opinable. As sight and the visible require light, the sun is introduced as the source of light. The analogical account of the status of the Good in the intelligible world is: “as the Good is in the intelligible region with respect to intelligence and the intelligible, so the sun is in the visible region with respect to sight and the visible” (508b13-c2).<sup>18</sup> Since the introduction of light and the sun is dependent on the specific use of sight and the visible as the only pair that require a third kind of thing, this analogical account is also dependent on the same specific use of sight and the visible. That is why, according to Ferguson, the visible is “purely a symbol of the intelligible” and cannot be “transmuted into the sensible or opinable”<sup>19</sup> in the Simile of the Sun.

After a short pause (509c1-11), Socrates begins the Simile of the Line by recalling that the Good is king of the intelligible region while the sun is king of the visible region (509d1-3). Thus the Simile of the Line takes over the visible and the intelligible from the Simile of the Sun, and it makes the lower segment AC of the line stand for the visible and the upper segment CB the intelligible (509d6-9). So, just as the visible was “purely a symbol of the intelligible” in the Simile of the Sun, it must be so in the Simile of the Line, too, so argues Ferguson.<sup>20</sup>

The purpose of the Simile of the Line, according to Ferguson, is to illustrate a distinction between CE and EB, which represent two stages of knowledge, within the

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<sup>17</sup> Apparently the existence of air as the medium for the transmission of sounds did not come to Plato’s mind (probably because we do not usually experience the absence of air). But this does not affect our project of interpreting Plato.

<sup>18</sup> Here too, despite the apparent structure of the sentence, Plato is not instructing us to understand the status of the sun in the visible world from the status of the Good in the intelligible world, but the other way around.

<sup>19</sup> Ferguson, “Plato’s Simile of Light: Part I,” 136.

<sup>20</sup> Ferguson, “Plato’s Simile of Light: Part I,” 136-7.

intelligible by the distinction between AD and DC within the visible.<sup>21</sup> There is no doubt that it is a function of the Simile of the Line to make a distinction between CE and EB within the intelligible. But it is not the sole function. Plato states that “with respect to truth or lack of it, as the opinable is distinguished from the knowable, so the likeness is distinguished from that of which it is the likeness” (510a8-10).<sup>22</sup> So it is one important function of the Simile of the Line to illustrate the relation between the opinable (i.e., the visible) and the knowable (i.e., the intelligible) by the relation between images and their originals.

To refute Ferguson’s interpretation completely, we need to go back to the visible in the Simile of the Sun. First of all, in the beginning of the Simile of the Sun, Plato, reminding us of the distinction he has made between the many F things and the F itself in Book V, tells us that the many F things are seen but not intellected while the F itself is intellected but not seen (507a7-b11).<sup>23</sup> The many F things are what is recognized and loved by the lovers of sight, lovers of hearing, lovers of crafts and practical people (476a10),<sup>24</sup> and they are what comes to be and perishes (485b2-3). So in the Book V

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<sup>21</sup> Ferguson, “Plato’s Simile of Light: Part I,” 136, and 143. I shall discuss the relation between AD and DC, and so between CE and EB with a certain limitation, in the next section.

<sup>22</sup> For the direction of the analogical explanation here, refer to notes 9 and 18 above. For the reason that “of the two pages given to this analogy [the Simile of the Line] in Stephanus’ edition less than a fifth suffices for the lower line,” Ferguson thinks that the meaning of AD and DC “seems to Plato to be so obvious as to need no discussion” (“Plato’s Simile of Light: Part I,” 137). Certainly Plato does not spend much space on the lower line; but this does not mean that it has no meaning to illustrate the relation between the visible and the intelligible by the relation between AD and DC. I think that the first part (509d1-510b1) of the Simile of the Line is a direct continuation and completion of the Simile of the Sun, whereas the second part (510b2-511e5) is a new topic Plato launches in the middle of the Simile of the Line to prepare the reader for the coming Simile of the Cave.

<sup>23</sup> “(T)he things stated here earlier” (507a7-8) refers to the discussion of the objects of knowledge and opinion in Book V.

<sup>24</sup> This kind of people also seem to be called “wine-lovers” (475a5), and “gain-loving” (581c4), i.e., loving of “eating, drinking, sex, and all their followers” (580e3-

discussion, what is seen stands for the sensible in general, and Plato begins the Simile of the Sun, having that concept in mind. This fact cannot be dismissed simply by saying that Plato “recalls this opposition [between the many F things and the F itself] purely in order to select a single element in matter.”<sup>25</sup> For the objects of opinion are in evidence in the midst of the Simile of the Sun, too. After explaining the status of the Good within the intelligible by analogy to the status of the sun in the visible, Plato contrasts what is seen under the light of the sun with what is seen under the light of night (508c4-d3); and the former represents the objects of knowledge while the latter represents the objects of opinion, what comes to be and perishes (408d4-10). So in terms of relative clarity or obscurity, as seeing the visible under the light of the sun is to seeing the visible under the light of night, so recognizing the intelligible under the Good is to recognizing the sensible under the sun.<sup>26</sup> Thus the sensible, too, is at issue in the Simile of the Sun.

Certainly the sensible in evidence here is what is symbolized rather than the symbol.<sup>27</sup> But the sensible is not limited to what is symbolized, for soon afterwards Plato says that “the sun not only provides what is seen with the power of being seen, but also with generation, growth, and nourishment” (509b2-4). What is provided with generation, growth, and nourishment by the sun cannot be merely visual appearances, but must be material things (animals and plants).<sup>28</sup> So the visible in the symbol is the sensible.

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<sup>25</sup> Ferguson, “Plato’s Simile of Light: Part I,” 133.

<sup>26</sup> This point is well discussed by H. L. Sinaiko, *Love, Knowledge and Discourse in Plato: Dialogue and Dialectic in Phaedrus, Republic, and Parmenides*, 126-31; except that strangely he thinks opinion is a medium like nocturnal light, when it is in reality a cognitive state as well as faculty. Cf. Murphy, “The ‘Simile of Light’,” 94, too.

<sup>27</sup> Cf. Raven, “Sun, Divided Line, and Cave,” 23.

<sup>28</sup> Later the visible of the Simile of the Sun is also referred to by “the bodily and visible” (532c7-d1).



Although Ferguson believes that Plato defines the visible and never changes its definition in the Simile of the Sun,<sup>29</sup> in fact Plato never defines the visible there. What Plato is doing is to pick up one aspect, i.e., visual appearances, of matter merely as a device for introducing light and the sun. Once introduced, the sun throws its light not only to colors but also to animals, plants, and other things around us.<sup>30</sup> Plato is concerned primarily with the visible as representative of the sensible throughout the Simile of the Sun.

If that is the case, the visible would mean the sensible in the Simile of the Line, too. This is well confirmed. First, the objects in DC are “the animals around us, and everything that grows, and the whole class of artifacts” (510a5-6) rather than merely colors.<sup>31</sup> So they differ from the objects in AD, which are shadows and reflections (509e1-510a3), not only in clarity (509d9) but also in truth or reality (510a9).<sup>32</sup> For animals and other things around us are tangible while shadows and reflections are not. Second, the visible is also called the opinable (510a9), and its cognition opinion (511d4). The opinable is the sensible particulars, which come to be and perish (485b2-3), and which are recognized and loved by the many (479d3).<sup>33</sup> So the relation between AD and DC within the visible, i.e., the sensible, symbolizes the relation between the sensible and the intelligible as well as the relation between CE and EB within the intelligible.

The final confirmation of our interpretation is the fact that the line in the Simile of the Line is one continuous line instead of two separate lines.<sup>34</sup> This is indeed the

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<sup>29</sup> Ferguson, “Plato’s Simile of Light: Part I,” 133.

<sup>30</sup> One can think of the whole natural environment as being ruled by the sun.

<sup>31</sup> Pace Raven, “Sun, Divided Line, and Cave,” 24, and *Plato’s Thought*, 148. As Raven admits (“Sun, Divided Line, and Cave,” 24, and *Plato’s Thought*, 149), originals are truer than images, even though the latter can be as clear as the former.

<sup>32</sup> Precisely speaking, the clarity of a cognition, for Plato, derives from the reality of its object (511e2-4).

<sup>33</sup> The many also believe that the good is pleasure (505b5-6).

<sup>34</sup> Cf. Cross & Woosley, *Plato’s Republic*, 212, and also Ross, *Plato’s Theory*, 68.

major difference between the Simile of the Sun and the Simile of the Line.<sup>35</sup> In the Simile of the Sun, though the visible region symbolized the intelligible region, they were two separate regions. The main function of the Simile of the Sun is to illustrate the role of the Good in the intelligible region by analogy to that of the sun in the visible region. The account of the defective state of cognition in each region (508c4-e3) may seem to suggest a certain connection between the two regions. The account, however, only serves the illustration by contrasting the defective state with the perfect one. This contrast amounts to saying that just as seeing the visible under the light of night is much less clear than seeing the visible under the light of the sun, so recognizing the opinable under the sun is much less clear than recognizing the intelligible under the Good. Excepting the point that the intelligible is now ruled by the Good, the contrast expresses nothing more than the idea, familiar from Book V, that knowledge is clearer than opinion. This idea does not yet tell us how the intelligible is connected with the opinable.

In the Simile of the Line, on the other hand, the intelligible and the visible are arranged on one continuous line. This suggests that there is a large category that includes both the intelligible and the visible, and hence the commonality between the two.<sup>36</sup> So, just like the length of each segment of the line, the reality of the objects in each segment is measured in a common scale (511e2-3).<sup>37</sup> Since this is so, the

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There is another point that suggests some connection between the visible and the intelligible. That is, the mathematicians use the objects in DC as images for recognizing the objects in CE (510e1-511a1). I shall discuss it later in connection with the equality of DC and CE.

<sup>35</sup> Another interesting difference between the two Similes is that the sun makes virtually no appearance in the Simile of the Line.

<sup>36</sup> Sinaiko calls this large category “being”, and its cognition “intelligence”, thinking that the visible (becoming) is only something imperfect of the intelligible (being). Cf. Sinaiko, *Love, Knowledge and Discourse*, 154-5.

<sup>37</sup> Literally translated, the text (511e2-4) reads: “arrange them [*noêsis, dianoia, pistis, eikasia*] in a proportion, and believe that as the *segments* to which they correspond *participate in truth*, so they participate in clarity” (emphasis added). Although it seems

intelligible can be related to the visible in a rather precise way through the mathematical proportion, which is explained by the originals and images within the visible. We can now discuss the relationship between AD and DC.

### 3. *Eikasia*

In this section I shall discuss the significance of *eikasia*. The problem is concisely put by Robinson: “whether by ‘conjecture’ or *eikasia* Plato meant trying to apprehend realities through images or taking images as themselves the realities to be apprehended.”<sup>38</sup> How to solve this problem will determine the relationship between AD and DC, which indicates the relationships between CE and EB as well as between AC and CB. My discussion of the problem here will be divided into three steps. First I shall critically examine the reasons for the first view of *eikasia* as indirect apprehension. Then I shall consider the reasons for the second view of *eikasia* as delusion, which is the view I take. Lastly I shall resuscitate some legitimate point of the first view with a certain qualification.

The reasons for the first view are threesome: the characteristics of mathematics in CE, the linguistic hint of *eikasia*, and the examples of objects of *eikasia*.<sup>39</sup> First Robinson, a proponent of the first view, seeks a clue to *eikasia* in *dianoia* since *eikasia*

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grammatically possible, it is not natural to take the antecedent of the relative clause “to which they correspond” as the objects. Cf. P. Shorey’s translation in E. Hamilton and H. Cairns, *The Collected Dialogues of Plato*, 747; and Murphy, “The ‘Simile of Light,’” 99, note 1. The antecedent of the relative clause should be taken as the segments. The segments are said to participate in truth because the length of the segments symbolizes the degree of truth or reality. The overall meaning of this text is that the reality of the objects and the clarity of their cognition in each segment correspond to the length of the segment.

<sup>38</sup> R. Robinson, *Plato’s Earlier Dialectic*, 190. “Conjecture” is Robinson’s translation of *eikasia*.

<sup>39</sup> Robinson, *Plato’s Earlier Dialectic*, 190-91; and also Ferguson, “Plato’s Simile of Light: Part I,” 144-5.

(AD) to *pistis* (DC) is *dianoia* (CE) to *noêsis* (EB). Now, according to Plato, there are two characteristics that separate mathematics in CE from philosophy<sup>40</sup> in EB (510b4-511d5). One characteristic (a) is that mathematicians use as images the visible things they draw, which belong to DC, for the sake of understanding those things that cannot be seen, whereas philosophers do not use any sensible images. The other characteristic (b) is that mathematicians go down from hypotheses, which they do not give any account of, whereas philosophers go up from hypotheses to the first beginning. Robinson thinks that since the characteristic (a) lends itself to *eikasia*, which is a state concerned with images, more naturally than the characteristic (b), *eikasia* is similar to mathematics in “attempting to know realities through images.”<sup>41</sup> Second, *eikazô*, the verb from which *eikasia* is derived, means: to represent by a likeness, to infer from comparison, to conjecture.<sup>42</sup> So *eikasia* would mean a state of trying to know originals through their images. Third, the examples Plato gives us as objects of *eikasia* are “shadows, then appearances produced in water and in all close-grained, smooth, bright things”(510a1-2). But we scarcely take those things for originals. So, argues Ferguson, those things would have been bad examples if Plato had meant by *eikasia* taking images as originals.

The three reasons above for the first view of *eikasia*, however, face the following difficulties. First, *eikasia* is not similar to mathematics as Ferguson contended. For, according to the first view, *eikasia* sees its *own* objects in AD to recognize their originals whereas mathematics recognizes its *own* objects in CE by seeing their images.<sup>43</sup> In other words, images are the objects of *eikasia*, but the objects of mathematics are originals rather than images. Second, although there is a natural linguistic connection between *eikasia* and *eikazô*, Plato’s use of *eikasia* does not depend on

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<sup>40</sup> Literally translated, dialectic, but by it Plato means philosophy.

<sup>41</sup> Robinson, *Plato’s Earlier Dialectic*, 191.

<sup>42</sup> Cf. Liddell & Scott, *Greek-English Lexicon*.

<sup>43</sup> Cf. W. F. R. Hardie, *A Study in Plato*, 62.

*eikazô*. As a matter of fact, Plato never uses *eikazô* in the Simile of the Line. Third, we scarcely try to know “the animals around us, and everything that grows, and the whole class of artifacts”(510a5-6) through the kind of examples Plato gives us as objects of *eikasia*.<sup>44</sup> So, by the same logic as before, those examples would have been bad examples if Plato had meant by *eikasia* trying to know originals through images.

Now the reasons for the second view of *eikasia* as delusion are the following. First, Plato’s use of *eikasia* is based on *eikonês* (images), which are the objects of *eikasia*, and *eikasia* means simply “the state of mind concerned with images.”<sup>45</sup> Second, by the examples Plato gives us as objects of *eikasia*, he refers to our not-so-uncommon experience of mistaking shadows and reflections for real animals and other things. Occasionally we do have this kind of experience although we realize our mistakes fairly quickly. This is an important point: we do make a mistake, and we well know that our mistake is a mistake. That is to say, we occasionally fall into *eikasia*, but we are not confined to it. Our ordinary condition is *pistis*, and from this vantage point we know well the nature of *eikasia*: how defective it is. This is the knowledge that enables us to understand by analogy what it is like to rise above the limitation of *pistis*, or *doxa* (opinion). Third, since the major point of the Simile of the Line is that *eikasia* (AD) to *pistis* (DC) is opinion (AC) to knowledge (CB), we may seek a clue to *eikasia* in opinion.<sup>46</sup> In Book V, Plato says that the sight-lovers, whose cognitive condition is opinion, are dreaming while the philosophers, whose cognitive condition is knowledge, are awake (476c2-4, c9-d4). He explains dreaming as “believing a likeness of something to be not a likeness, but rather the thing itself to which it is like”(476c6-7). If opinion, in comparison with knowledge, is like dreaming, so would *eikasia* be in comparison with *pistis*, and *eikasia* would be taking images for originals.<sup>47</sup> Fourth, a

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<sup>44</sup> The only common experience of this kind of indirect cognition is looking at oneself in the mirror.

<sup>45</sup> Cross & Woosley, *Plato’s Republic*, 218.

<sup>46</sup> Cf. Hardie, *A Study in Plato*, 58, and Cross & Woosley, *Plato’s Republic*, 219.

similar point can be made about mathematics in CE in comparison with philosophy in EB.<sup>48</sup> In Book VII, Plato says that though concerned with the intelligible, mathematical sciences<sup>49</sup> “dream about what is”(533b8-c1). The reason for this is related with the second characteristic of mathematics: the mathematical sciences “haven’t the capacity to see it [what is] in full awakesness so long as they use hypotheses and, leaving them untouched, are unable to give an account of them” (533c1-3). Mathematicians leave hypotheses untouched because they believe their hypotheses, which look like the beginning, to be the beginnings themselves (510c2-d1).<sup>50</sup> So mathematicians are in a dreaming condition. If so, *eikasia* would be a dreaming condition, too.

Those are, I believe, convincing reasons for taking the second view of *eikasia* as delusion. What is tricky about delusion, however, is that people who have a delusion do not believe that they have a delusion. As Hardie noted,<sup>51</sup> the sight-lovers do not believe in their own words that “this thing, which is a likeness of something else, is not a likeness, but that thing itself to which it is like.” It is self-contradictory for anybody to believe that “this thing is a likeness of something else and not a likeness of that.” The central point of delusion is that people who have a delusion do not know that they have a delusion. Delusion is a description of somebody’s cognitive condition from a third person point of view.<sup>52</sup> From their own first person point of view, the sight-lovers do

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<sup>47</sup> If *eikasia* were indirect cognition, so would opinion be. But the sight-lovers do not admit that there is the knowable of which the opinable is a likeness (476b4-9, c2-4, 479a1-5). Therefore *eikasia* cannot be indirect cognition.

<sup>48</sup> Cf. Hardie, *A Study in Plato*, 59-60, and Cross & Woosley, *Plato’s Republic*, 219.

<sup>49</sup> The mathematical sciences include arithmetic (522c5-526c8), plane geometry (526c8-527c11), solid geometry (528a9-e2), astronomy (528e3-530c5), and harmonics (530c5-531c8).

<sup>50</sup> The relation between the two characteristics of mathematics (the use of sensible images, and the use of hypotheses) is a difficult matter. It seems to me, though, that the use of sensible images is more fundamental. I will discuss this problem in a paper I intend to write on the mathematician’s hypothetical method and the philosopher’s dialectical method.

<sup>51</sup> Hardie, *A Study in Plato*, 58.

not have a delusion: they are simply convinced that what they see is real. But from a point of view of a third person with a better understanding, what the sight-lovers see is in fact a likeness of the knowable; and they have a delusion in taking the opinable, which is only an image, for the reality itself. Also from this third person point of view, insofar as what the sight-lovers see is in fact a likeness of the knowable, they are recognizing something indirectly about the knowable. So this is what I believe is a legitimate point of the first view of *eikasia* as indirect cognition: from a point of view of somebody in DC, we can say that people who have *eikasia* are trying to know originals through images, even though they are not aware of it.<sup>53</sup>

In assessing the sight-lovers' cognition, it is hard for us in DC to imagine what a point of view of a third person with a better understanding is like. That is why Plato brings in a quite familiar analogy of *eikasia* and *pistis*: how *eikasia* is seen from a third person point of view of *pistis*.

#### 4. The Equality of DC and CE

The third major interpretative problem I want to discuss is the equality of DC and CE. First, I shall present the problem. Since, according to Plato's directions on the

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<sup>52</sup> Cf. Ross, *Plato's Theory*, 67, where he writes that *eikasia* as Plato uses it is not "a consciously insecure attitude towards its objects," but "*eikasia* and *pistis* as used here by Plato are distinguished . . . by a smaller or greater *actual* security in their grasp of reality" (emphasis added).

<sup>53</sup> It is often thought a mystery why Plato tends to suggest that the objects of the sight-lovers' cognition is different from those of the philosophers' cognition rather than that through what the sight-lovers directly recognize, they indirectly recognize the same objects that the philosophers directly recognize. Plato is respecting the sight-lovers' viewpoint that there is nothing beyond what they see. Indeed, it would not be fair to attribute to the sight-lovers the view that they are recognizing the knowable through the opinable, at least until they are converted. Plato is also emphasizing the cognitive wretchedness of the sight-lovers, who cannot even start any progress, because they would not admit that they do not know the knowable.

division of the line, a line is divided into two unequal segments, and each segment is divided in the same ratio (509d6-8), the middle two segments, DC and CE, come out equal.<sup>54</sup> But, on the other hand, the relative length of segments represents the relative truth of objects, and the relative clarity of cognitions (509d8-510a10, 511d6-e4). And *dianoia* is between *pistis* and *noêsis*, that is, clearer than *pistis* but less clear than *noêsis* (511d4-5). Further since mathematicians, whose cognitive condition is *dianoia*, use the visible things as images for the sake of understanding the intelligible (510d5-511a1), the same image-original relationship holds between the objects in DC and those in CE, as well as between the objects in AD and those in DC and between the objects in CE and those in EB. These considerations suggest that the four cognitive states, *eikasia*, *pistis*, *dianoia*, and *noêsis*, are arranged so that each state is surpassed by the next one in clarity. So, it seems, if DC and CE are equal, *pistis* and *dianoia* should be equally clear; but if *dianoia* is clearer than *pistis*, CE should be longer than DC. In other words, the equality of DC and CE contradicts Plato's assertion that *dianoia* is clearer than *pistis*.

Facing this problem, most scholars think that the equality of DC and CE is an unintended consequence of the mechanism of the division of the line.<sup>55</sup> According to this orthodox view, what Plato wanted to convey through the division of the line is the equality of the proportions he explicitly mentions, between AC and CB, between AD

<sup>54</sup> To prove this is easy. Suppose that  $AB = 1$ , and  $AC = t$ . Then  $CB = 1-t$ .  $DC = AC \times (CB/AB) = t \times ((1-t)/1) = t \times (1-t)$ .  $CE = CB \times (AC/AB) = (1-t) \times (t/1) = (1-t) \times t$ . Now  $t \times (1-t) = (1-t) \times t$ . Therefore  $DC = CE$ .

<sup>55</sup> Adam, *The Republic of Plato* Vol. II, 64; Hardie, *A Study in Plato*, 56; Ross, *Plato's Theory*, 45-6; Cross & Woosley, *Plato's Republic*, 204; Raven, *Plato's Thought*, 145. The same view seems to be shared by Guthrie, *A History of Greek Philosophy* IV, 508, and J. Annas, *An Introduction to Plato's Republic*, 247-9. Ferguson and Murphy hold the same view for different reasons. For Ferguson, DC is compared with AD within the visible while CE is compared with EB within the intelligible, but there is no way of comparison that crosses the border between the visible and the intelligible ("Plato's Simile of Light: Part I," 138, note 3). For Murphy, the line is divided into equal segments, and the lengths of segments do not matter at all ("The 'Simile of Light'," 99, especially note 1, and *The Interpretation*, 158-9). Concerning Murphy's view, see also note 6 above.



and DC, and between CE and EB;<sup>56</sup> and the equality of DC and CE, as his silence about it testifies, was not intended to be taken as significant. But in spite of the equality of DC and CE, Plato wanted *dianoia* to be clearer than *pistis*; for *dianoia* is related to the intelligible whereas *pistis* is related to the visible, and *dianoia* is located above *pistis* on the line.<sup>57</sup> Because the orthodox view is the simplest solution of the problem, I believe it is the right one. And being the minimalist solution of the problem, the orthodox view would require the least defense. However, there are two other bolder views, which respectively finds, and seems to find, a meaning in the equality of DC and CE. In the rest of this section, I shall critically examine those two views as a way of defense of the orthodox view.

First, according to Brumbaugh, “Plato deliberately put this discrepancy [between the metaphor of proportion and the metaphor of inequality] into his text, or at least deliberately allowed it to remain there.”<sup>58</sup> On the one hand, the metaphor of proportion expresses the analogical relations between different types of cognition: opinion : knowledge = *eikasia* : *pistis* = *dianoia* : *noêsis*. It underscores the similarities between those levels of cognition. On the other hand, the metaphor of inequality brings out the differences in clarity and adequacy between those levels of cognition by differentiating lengths of segments. For the metaphor of inequality, Brumbaugh proposes the division of a line AB such that EB = 1, CE = 2, DC = 4, and AD = 8.<sup>59</sup> In the line thus

<sup>56</sup> Later, Plato also mentions the same proportion between *noêsis* and *pistis*, and between *dianoia* and *eikasia* (533e7-534a5). This fact suggests that Plato was at least aware of the equality of DC and CE.

<sup>57</sup> The line is a vertical line with B being the top and A being the bottom, and the four cognitive states that correspond to the four segments of the line are *noêsis*, *dianoia*, *pistis*, and *eikasia* from the top (511d8-e2, 511a7, 517b5).

<sup>58</sup> R. S. Brumbaugh, “Plato’s Divided Line,” 529-30. See also his *Plato’s Mathematical Imagination: The Mathematical Passages in the Dialogues and Their Interpretation*, 98-104.

<sup>59</sup> Brumbaugh, “Plato’s Divided Line,” 531, and *Plato’s Mathematical Imagination*, 102-4. For his proposal, Brumbaugh refers to Aristotle, *De Anima* 404b22-4, and Plato, *Epinomis* 991a1-4.

divided into unequal segments,  $AD : DC = DC : CE = CE : EB$ .<sup>60</sup> Now it is impossible to construct a single line that incorporates both the metaphor of proportion and the metaphor of inequality. It is, according to Brumbaugh, a warning Plato gives to the reader “against taking mathematical images too literally.”<sup>61</sup> Consequently, “the reader should have two distinct diagrams, but be mistrustful of both.”<sup>62</sup>

There are two elements in Brumbaugh’s view. The first one is, of course, that the discrepancy between the metaphor of proportion and the metaphor of inequality is intentional on Plato’s part. The second one is that the length of segments represents the distance between the cognitive agent and the object of cognition: the longer the segment is, the less clear the cognition is.<sup>63</sup> Thus EB is the shortest, and AD is the longest segment.

Brumbaugh’s second point is quite unique as it is in opposition to almost all interpreters’ view.<sup>64</sup> But his point seems insignificant in a sense. Plato does not explicitly mention any of the segments as the longer or the shorter. Whichever, EB or AD, is the shortest, the metaphor of inequality would be the same:  $AD : DC = DC : CE = CE : EB$ .<sup>65</sup> If EB is the shortest, then the shortest segment will represent the clearest cognition; and if EB is the longest, then the longest segment will represent the clearest cognition. That is all the difference. However, Brumbaugh’s idea of “the shorter, the better” does not fit Plato’s metaphorical use of language. Plato does not say that the

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<sup>60</sup> As Brumbaugh notes (“Plato’s Divided Line,” 531), this division of the line also satisfies Plato’s later description of the line (534a4-5): knowledge : opinion = *noêsis* : *pistis* = *dianoia* : *eikasia* ( $3 : 12 = 1 : 4 = 2 : 8$ ).

<sup>61</sup> Brumbaugh, “Plato’s Divided Line,” 532-3.

<sup>62</sup> Brumbaugh, “Plato’s Divided Line,” 534.

<sup>63</sup> Brumbaugh, *Plato’s Mathematical Imagination*, 99.

<sup>64</sup> According to Murphy, a rare exception on this matter is Plutarch, who thought that the lower segment is the longer “because it contains the many, while the upper contains the one” (“The ‘Simile of Light’,” 99, note 1).

<sup>65</sup> Similarly, the metaphor of proportion would be the same:  $AC : CB = AD : DC = CE : EB$ .

longer the distance between the cognitive agent and the object of cognition is, the less clear the cognition is. In Book V, knowledge and opinion were defined within the framework of the two extreme terms, what is and what is not, the latter of which is nothing (478e4-5, 478b12-c1). There the *more* the object partakes of reality, the *clearer* the cognition is (479c8-d1). In the Simile of the Line, too, “as the segments to which they [cognitive states] correspond participate in truth, so they participate in clarity” (511e2-4). This means that the *more* the segments participate in truth, the *more* the cognitive states participate in clarity. It is quite unnatural for the shorter segment to represent the more participation in truth. The more participation (of objects) in truth can only be represented by the longer segment. Certainly the smaller number can represent the object that participate the more in truth, but Plato does not mention any number in the Simile of the Line. So Plato’s metaphor is not one of number, distance and unclarity, but one of length, truth and clarity.<sup>66</sup>

As for the first point Brumbaugh has made, the metaphors of proportion and inequality are not so different as he suggests. For the metaphor of proportion at once expresses both the difference and the connection between different types of cognition; while the metaphor of inequality also involves proportions between the four levels of cognition, as it is based on the image-original relationship between AD and DC, between CE and EB, and between DC and CE. So, it is better to say, those metaphors are two different metaphors of proportion: the first one is  $AD : DC = CE : EB = AC : CB$ , and the second one is  $AD : DC = CE : EB = DC : CE$ .

Brumbaugh’s central contention is that the discrepancy between the two metaphors is intentional. His reason for it is that all the key relations of the four levels of cognition cannot be adequately represented by spatialization such as a line.<sup>67</sup> This is a sound point. The characteristics of Forms Plato wanted to convey are “trans-spatial,

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<sup>66</sup> Cf. Adam, *The Republic of Plato* Vol. II, 64.

<sup>67</sup> Brumbaugh, “Plato’s Divided Line,” 533-4, and *Plato’s Mathematical Imagination*, 98.

trans-temporal,”<sup>68</sup> but the means which he had to adopt for doing so was spatial. So his means are intrinsically inadequate for his purpose. This sound point, however, is sometimes expressed by an interpreter holding the orthodox view, too; for example, Ross has written, “the line, being but a symbol, is inadequate to the whole truth which Plato meant to symbolize.”<sup>69</sup> The point in dispute is rather: can we say that Plato intended the equality of DC and CE? Certainly Plato *intended* the first metaphor of proportion to express part of the truth he meant, but is the equality of DC and CE a significant part of the metaphor? Here Brumbaugh thinks that Plato intended the equality of DC and CE to be a significant part of the first metaphor of proportion, the first metaphor thereby to contradict the second metaphor of proportion, and the reader to realize the contradiction and the inadequacy of the metaphors. Although Brumbaugh’s view contains a sound point that the symbol is not adequate for what is symbolized, to say that Plato intended the reader to realize that point, is far-fetched. For the present discussion in the Simile of the Line is concerned with the intelligible and the visible, and not with the Simile itself. Also there is no clear suggestion in the text that the two metaphors contradict, and so it is quite possible for the reader not to notice the contradiction. Therefore it is safer to conclude that the equality between DC and CE is not intended.

The other bold view, which is proposed by N. Cooper and N. White,<sup>70</sup> seems<sup>71</sup>

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<sup>68</sup> Brumbaugh, *Plato’s Mathematical Imagination*, 98.

<sup>69</sup> Ross, *Plato’s Theory*, 46.

<sup>70</sup> Cf. N. Cooper, “The Importance of *dianoia* in Plato’s Theory of Forms,” 65-9; and more importantly N. White, *A Companion to Plato’s Republic*, 184-6, and also *Plato on Knowledge and Reality*, 96-8, especially 110, note 34. Cooper’s view seems the same as White’s although Cooper tends to express his view by saying that *dianoia* is an indirect cognition of Forms. J. Klein (*A Commentary on Plato’s Meno*, 112-25) holds a view similar but peculiar in its own way.

<sup>71</sup> I have to say “seems”, because it is my interpretation of Cooper’s and White’s view. They do not mention the equality of DC and CE in the expositions of their view, although White hints at it.

to find a meaning in the equality of DC and CE. This time the meaning of the equality is that the objects in DC and those in CE are the same, i.e., the sensible particulars. The difference, however, between DC and CE is one of aspect. In DC, the sensible objects are seen in themselves or as the things, of which the shadows and reflections in AD are images, but in CE the same sensible objects are seen as images of their originals, i.e., of the Forms in EB.<sup>72</sup> According to this view, there are three kinds of objects: first Forms in EB, second the sensible particulars in DC, which are images of Forms, and third shadows and reflections in AD, which are images of the sensible particulars. But there are four kinds of cognitive states; first *noêsis* recognizes Forms in themselves; second *dianoia* recognizes Forms in their sensible images; third *pistis* recognizes the sensible particulars in themselves; and fourth *eikasia* recognizes the sensible particulars in their images.<sup>73</sup> This is a very neat interpretation. The difference in number between the kinds of objects and the kinds of cognitive states is explained by the fact that one kind of objects play a double role: once in *pistis* as originals and once in *dianoia* as images. Thus this view is able to do two difficult tasks at once. First it can explain why CE is part of CB: *dianoia* recognizes Forms, though indirectly in sensible particulars. Second it can explain why DC and CE are of the same length: the objects directly recognized by *dianoia* are the same as those recognized by *pistis*.<sup>74</sup>

This view, however, cannot be maintained for the following reasons. First, the biggest feature of the view is that *dianoia* does not have its own proper objects.<sup>75</sup> It is

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<sup>72</sup> White, *A Companion to Plato's Republic*, 185. It is generally agreed that the objects in EB are Forms (510b8-9, 511c1-2).

<sup>73</sup> Cooper, "The Importance of *dianoia*," 67. Concerning *eikasia*, White differs from Cooper in thinking that *eikasia* is recognizing nothing more than images (*A Companion to Plato's Republic*, 185-6). In this respect, Cooper is more consistent than White since *noêsis* : *dianoia* = *pistis* : *eikasia*.

<sup>74</sup> This view can further suggest that *dianoia*, though a cognition of Forms, is only as clear as *pistis* because it recognizes them only in the sensible particulars (511e2-4); however, it is suggested neither by White nor by Cooper.

<sup>75</sup> According to White, the division of CB into CE and EB is not a division of objects,

like hanging in the air between the sensible particulars and Forms, and yet being related to both of them in some way or another. But the line represents objects (509d6-8), and the division of the line a division of the objects (510b2, 511c3-6, 534a5-7). Further, in Plato's way of thinking, there is a one-to-one correlation between the objects and the cognitive states: the cognitive states differ if and only if the objects of cognition differ (477c9-d5).<sup>76</sup> So if *dianoia* is a distinct cognitive state different both from *noêsis* and *pistis* (as it apparently seems so),<sup>77</sup> then the objects of *dianoia*, too, should be different both from those of *noêsis* and those of *pistis*. Thus it is untenable to say, as White does, that "the objects with which geometry deals are just sensible objects."<sup>78</sup> As CE is part of CB, the objects geometry deals with must be intelligible objects.<sup>79</sup> Second, Cooper and White take *dianoia* as indirect cognition. But the view of *eikasia*, and so of *dianoia*, as indirect cognition has been already criticized in the section 3 above. Third, the Forms, which are the objects in EB, are the things, the existence of which the many do not admit (476c2-4, 479a1-3). But the mathematicians regard the objects<sup>80</sup> they deal with as obvious to everyone (510d1). So the objects of *dianoia* are not the same as the Forms, either.<sup>81</sup> Although the bold view examined here suggests that the objects of

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and if it is, the objects of *dianoia* overlap those of *noêsis*, and so no special objects are required for CE (*Plato on Knowledge*, 110, note 32). But his view is quite inadequate as an account of Plato's text.

<sup>76</sup> This is clearly implied by Plato's principle of individuation for faculties, which states that faculties are the same when they are related to the same objects and produce the same cognitive states, and that faculties are different when they are related to different objects and produce different cognitive states (477d2-5).

<sup>77</sup> The faculty that produces *noêsis* is called dialectic (511b4) and *noêsis* (532b1) while the one that produces *dianoia* is called *dianoia* (511c7). Also the science that deals with the objects of *noêsis* is called dialectic (533c7) while the sciences that deal with the objects of *dianoia* are called mathematical arts (533b7-8).

<sup>78</sup> White, *Plato on Knowledge*, 110, note 34.

<sup>79</sup> The sensible objects are used by the mathematicians only as an aid for thinking about the intelligible (510d5-511a1). Cf. Cross & Woosley, *Plato's Republic*, 233 and 238.

<sup>80</sup> For example, the odd, the even, the figures, and three kinds of angles (510c4-5).

*dianoia* are both sensible particulars and Forms at once, Plato's text indicates that it is neither of them. It is time to discuss the objects of *dianoia*.

## 5. The Objects of *Dianoia*

The last and biggest interpretative problem I want to discuss in the present paper is the objects of *dianoia*. Concerning this problem, there are two types of interpretation: the traditional interpretation holds that the objects of *dianoia* are mathematical intermediates,<sup>82</sup> while the modern interpretation holds that the objects of *dianoia* are Forms.<sup>83</sup> In this section, first I shall present the traditional interpretation, explaining what mathematical intermediates are. Then I shall discuss Cornford's view and Cross and Woozley's view as two representative versions of the modern interpretation, and I shall show the inadequacy of their views. That will make the traditional interpretation a plausible option. The modern interpretations were initially proposed, however, because the traditional interpretation seemed inadequate. So lastly, I shall remove the difficulties

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<sup>81</sup> Without regard to whether the mathematicians recognize their objects directly or indirectly.

<sup>82</sup> Adam, *The Republic of Plato* vol. II, 68-9, 114-16, 159-63, and Hardie, *A Study in Plato*, 49-55. J. Brentlinger ("The Divided Line and Plato's 'Theory of Intermediates'," 146-66), Raven (*Plato's Thought*, 154-9), Sinaiko (*Love, Knowledge and Discourse*, 148, 158-64), and M. F. Burnyeat ("Platonism and Mathematics: A Prelude to Discussion," 213-40) seem to agree with the traditional interpretation; though Raven (*Plato's Thought*, 156-9) and Sinaiko (*Love, Knowledge and Discourse*, 159-60) think that the objects of *dianoia* include more than mathematical intermediates. For Guthrie's and Annas' views, see the next note.

<sup>83</sup> F. M. Cornford, "Mathematics and Dialectic in the *Republic* VI-VII," 61-95, and Cross & Woozley, *Plato's Republic*, 230-38. J. C. Wilson ("On the Platonist Doctrine of *asumblêtoi arithmoi*," 247-60), Ross (*Plato's Theory*, 58-67), and Robinson (*Plato's Earlier Dialectic*, 192-201) take similar views. Guthrie (*A History of Greek Philosophy* IV, 523) and Annas ("On the 'Intermediates'," 160-64, and *An Introduction to Plato's Republic*, 251) have a little ambiguous view, admitting mathematical intermediates in the *Republic* Book VII but not in the Simile of the Line in Book VI.

felt about the traditional interpretation, giving a reasonable account of the passages in Plato's text which seem to pose those difficulties. Although here I cannot give a new, positive argument for the theory of mathematical intermediates, I have given such an argument elsewhere.<sup>84</sup>

First of all, the subdivision of the intelligible is a division of objects (510b2, 511c3-6, 534a5-7). Nobody doubts that Forms are intelligible objects (507b5-10) and that the objects of *noêsis* are Forms (510b8-9, 511c1-2). The objects of *dianoia* are originals of the sensible objects in DC, and images of Forms in EB.<sup>85</sup> The mathematicians' cognitive state is *dianoia*, and the objects of mathematics are objects of *dianoia*.<sup>86</sup> Further *dianoia* is "something between (*metaxu*) opinion and *noêsis*" (511d4-5).<sup>87</sup> This is only because the objects of *dianoia* are something between the objects of opinion and those of *noêsis*.<sup>88</sup>

Now Aristotle tells us about Plato's theory of intermediates as follows:

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<sup>84</sup> I have written a paper that defends the theory of mathematical intermediates as something inherent in the logical structure that generates Plato's Forms. Cf. my "Two Arguments for Forms: Conflicting Appearances and One over Many."

<sup>85</sup> The proportion that  $AD : DC = CE : EB$  implies that the image-original relationship holds between the objects in CE and those in EB.

<sup>86</sup> It is an open question whether the mathematicians are the only people who have *dianoia* and the objects of mathematics constitute the whole objects of *dianoia*, or there are other people who have *dianoia* and the objects of *dianoia* include objects other than those of mathematics. However, it would be safer here to regard the former view as true, on the ground that whatever else might be included, at least the objects of mathematics are included in the objects of *dianoia*.

<sup>87</sup> "[B]righter than opinion but dimmer than *noêsis*" (533d5-6). On the line, CE is between AC and EB, i.e., above AC but below EB.

<sup>88</sup> The nature of a cognitive state is dependent on the nature of its objects (477a2-b1, 477c9-d5). We may recall that in Book V opinion is defined as something between (*metaxu*) knowledge and ignorance because the objects of opinion are something between (*metaxu*) the objects of knowledge and those of ignorance (477a6-b1, 478e1-5). As something intermediate between the objects of knowledge and those of ignorance, the objects of opinion share something, i.e., being with the former and something else, i.e., not-being with the latter (477a6, 478e1-2).



besides sensible things and Forms he [Plato] says there are the objects of mathematics, which occupy an intermediate (*metaxu*) position, differing from sensible things in being eternal and unchangeable, from Forms in that there are many alike, while the Form itself is in each case unique.<sup>89</sup> (*Metaphysics* 987b14-18)

According to this report, Plato introduced the objects of mathematics as well as mathematical Forms, in addition to the sensible objects of which mathematical terms are predicated; for example, mathematical triangles as well as the Form of Triangle, in addition to visible triangles drawn on paper. The objects of mathematics are something intermediate between the sensible objects and Forms. For they differ from sensible objects in being eternal and unchangeable, i.e., share the eternity and unchangeability with Forms on the one hand, and differ from Forms in being many, i.e., share the plurality with the sensible objects on the other hand.

As Plato states, the mathematicians' thought is concerned not with sensible objects but with intelligible objects (510d5-511a1).<sup>90</sup> For the truth of mathematics, e.g., the Pythagorean proposition, would be true even if there is no sensible object that exemplifies that truth.<sup>91</sup> Now it is clear that Plato's Forms are each unique (476a2-7, 507b5-7). But, for example, when the geometrician says that one isosceles right triangle A is divided into two equal isosceles right triangles B and C, it cannot be about the Form of Triangle, because it involves three triangles while the Form is only one.

The same thing is true for numbers. When the arithmetician states that  $2+2=4$ , it cannot be about the Form of Two; for if "2" meant the Form of Two, which is unique, it would be nonsense to add it to itself. " $2+2=4$ " presupposes that there are two 2's that can be added to make 4. Explaining a Platonic doctrine that mathematical

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<sup>89</sup> The translation of Aristotle's *Metaphysics* I shall use is Ross's translation in J. Barnes, *The Complete Works of Aristotle*, Vol. 2.

<sup>90</sup> A little later in Book VII, Plato says that the geometrical knowledge is concerned with what is always, and not at all what is at any time coming into being and passing away (527b5-8).

<sup>91</sup> Either nothing looks like a right triangle, or what looks like a right triangle is not really so in that its sides are not perfectly straight, etc.

numbers are combinable, Aristotle says:

in mathematical number no unit is in any way different from another.  
(*Metaphysics* 1080a22-3)

According to this report, each mathematical number consists of units that are equal to each other, and that is why addition, subtraction, multiplication, and division of numbers are possible. By contrast, numerical Forms, which are each simple and do not consist of parts,<sup>92</sup> would not be subject to the arithmetical processes (or if they are, their processes would be very different from what we intuitively conceive as arithmetical processes).<sup>93</sup> Thus the doctrine of mathematical numbers, as Aristotle reports it, implies that there are many equal units, of which numbers consist, and that there are many mathematical numbers, 2's, 3's, etc., which consist of those units.<sup>94</sup>

That is what Aristotle tells mathematical intermediates are. Our problem of determining if the objects of mathematics are intermediates hangs on how explicit Plato is in saying that they are in each case many. First, since the image-original relationship holds between the objects of mathematics and Forms, it is only natural that there are many mathematical objects that correspond to each mathematical Form.<sup>95</sup> Further in Book VII, when Plato explains how arithmetic leads one's soul from becoming to being,

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<sup>92</sup> Forms are said to be monads (*Philebus* 15b1) and indivisible (*Timaeus* 35a1, a5).

<sup>93</sup> About numerical Forms that are each different in kind and non-combinable with any other, Aristotle adds that they have "before and after" (*Metaphysics* 1080b12). It would be interesting to compare mathematical numbers and numerical Forms with Russell's concept of (cardinal) numbers and Benacerraf's concept of (ordinal) numbers. Cf. B. Russell, *Introduction to Mathematical Philosophy*, 11-19, and P. Benacerraf, "What Numbers Could Not Be," 272-94. According to Russell, every 2 consists of two things; while according to Benacerraf, 2 does not consist of two things, but is defined in terms of its sequential relations with 1 and 3.

<sup>94</sup> Since each unit is numerically different, 2 that consists of unit A and unit B would be numerically different from 2 that consists of unit C and unit D.

<sup>95</sup> In the *Phaedo*, there are expressions "the equals themselves" (74c1) as well as "the equal itself" (74a12). The former may refer to intermediate objects, though the evidence is not conclusive.

he says that arithmetic is concerned with:

numbers . . . in which the one is as your axiom claims it to be . . . each one equal to every other one, without the slightest difference between them, and containing no parts within itself. (526a2-4)

This is exactly what we heard from Aristotle. It means that mathematical numbers consist of equal units, and implies that there are many units and many 2's, 3's, etc. From these reasons combined with the above mentioned understanding of the objects of *dianoia* as something between the objects of *noêsis* and those of *pistis*, the traditional interpretation concludes that the objects of *dianoia* are mathematical intermediates.<sup>96</sup>

Next, I shall discuss two modern interpretations. They do not admit mathematical intermediates in the *Republic* Books VI-VII. When no third class of objects is introduced between Forms and sensible particulars, the only kind of intelligible objects are Forms.<sup>97</sup> According to Cornford, the objects of *dianoia* are mathematical Forms such as the Square itself and the Diagonal itself (510d7-8), while the objects of *noêsis* are moral Forms such as the Beautiful itself and the Good itself (507b5).<sup>98</sup> The distinction between mathematical Forms and moral Forms, however, is not a difference of higher and lower classes of Forms, but it is only "a matter of expediency in teaching."<sup>99</sup> There are visible images of mathematical Forms; and such images are used by the mathematicians as an aid in their study (510b4, d5, 511a6-8). On the other hand there are no visible images of moral Forms. Needless to say, all Forms are invisible (507b9-10). So it is easier to distinguish mathematical Forms from their images than moral Forms from their images. "Accordingly," Cornford says, "mathematics serves as the easiest bridge from the sense world to the intelligible, and should precede the study of moral Ideas [Forms]."<sup>100</sup>

<sup>96</sup> Adam, *The Republic of Plato*, 159-60, and Hardie, *A Study in Plato*, 52-3.

<sup>97</sup> Cf. Cross & Woosley, *Plato's Republic*, 237.

<sup>98</sup> Cornford, "Mathematics and Dialectic," 62. Cornford thinks that mathematical and moral Forms are the whole content of the intelligible realm.

<sup>99</sup> Cornford, "Mathematics and Dialectic," 63.

But Cornford's view is not adequate for the following three reasons.<sup>101</sup> First, it is not the case that there are no visible images of moral Forms. The reason why Cornford thinks there are no visible images of moral Forms is that images of moral Forms are "invisible properties of souls."<sup>102</sup> Certainly moral characters of souls are not visible; but it is not because they are images of moral Forms but because they are characters of souls. There are other visible images of moral Forms. For example, just actions such as paying a tax and punishing a criminal are visible. Plato clearly states that many beautiful things and many good things are visible (507b2-9). And images of Beauty include "fair sounds and colors and shapes and all that craft makes from such things" (476b5-6).<sup>103</sup> Consequently, it is not harder to distinguish moral Forms from their images than mathematical Forms from their images. In both cases, Forms are not visible while their images are visible. So it is equally easy, if it is easy, to distinguish mathematical and moral Forms from their images (that is, insofar as one can recognize Forms).

Second, let us grant for the sake of argument Cornford's assertion that images of moral Forms are not visible. Then there is certainly a difference between images of mathematical Forms and those of moral Forms: the former are visible while the latter are invisible. So it is hard to distinguish moral Forms from their images because both of them are invisible. But it is easier, Cornford thinks, to distinguish mathematical Forms from their images because the former are invisible while the latter are visible. Cornford can say this only to those people who already recognize mathematical Forms. The fact that mathematical Forms are invisible while their images are visible, would help those people confirm their view on mathematical Forms. But the fact does not lead people

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<sup>100</sup> Cornford, "Mathematics and Dialectic," 63.

<sup>101</sup> For the criticism of the modern interpretation, especially Ross's, Robinson's and Cornford's views, refer to Brentlinger, "The Divided Line," 150-55.

<sup>102</sup> Cornford, "Mathematics and Dialectic," 62-3.

<sup>103</sup> Plato also suggests that moral dispositions can be represented by visible images (401b1-7).

who do not recognize mathematical Forms to the cognition of mathematical Forms. For the very fact, these people would deny mathematical Forms, while admitting that there are visible images of them, e.g., visible triangles drawn on paper. So the fact does not explain why mathematics “serves as the easiest bridge from the sense world to the intelligible.”

Third, Cornford simply ignores Plato’s point that the objects of *dianoia* are images of the objects of *noêsis*. For Plato, the image-original is a primary ontological distinction by which he understands the visible and the intelligible, and the objects of *dianoia* and the objects of *noêsis*. Though Cornford admits that the distinction between CE and EB is a distinction between the objects of *dianoia* and *noêsis*, he “denies that the distinction amounts to an ontological one.”<sup>104</sup> Thus in Cornford’s view, the segment CB of the line does not need to be a vertical one. And, as Cornford says, moral Forms do not need to be above mathematical Forms.<sup>105</sup> But this flatly contradicts Plato’s directions on the line. For Plato states that EB is the uppermost segment of the line (511d8), and by the mathematical proportion among the segments of the line (509d6-8), he means that EB is longer than CE, indicating that the objects in EB participate in more reality than the objects in CE (511e2-4).

Another modern interpretation is Cross and Woosley’s view. They take more seriously the image-original relationship that holds between the objects of *dianoia* and *noêsis*.<sup>106</sup> The image-original relationship implies that the objects of *dianoia* and *noêsis* are called, not by different names such as mathematical ones and moral ones,<sup>107</sup> but by the same names. Thus Cross and Woosley think that both the objects of *dianoia*

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<sup>104</sup> Brentlinger, “The Divided Line,” 154.

<sup>105</sup> Cornford, “Mathematics and Dialectic,” 62. Cornford’s interpretation of *dianoia* and *noêsis* is far more complicated by his exposition of their procedures (rather than objects); but his exposition of the procedures simply annihilates the distinction between the objects of *dianoia* and *noêsis* (“Mathematics and Dialectic,” 63-77).

<sup>106</sup> Cross & Woosley, *Plato’s Republic*, 219 and 259-60.

<sup>107</sup> Unless one can prove that mathematical Forms are images of moral Forms.

and *noêsis* are Forms in general.<sup>108</sup> The difference is: according to their view, the objects of *dianoia* are “Forms not seen in their connection with the Form of the Good,” while the objects of *noêsis* are “the Form of the Good and the other Forms seen in their connection with it.”<sup>109</sup> For example, the mathematicians, since they fail to go up from their hypotheses to the unhypothetical first principle, i.e., the Form of the Good (511b6-7, 533c1-3),<sup>110</sup> see mathematical Forms as “separate and unconnected.”<sup>111</sup> By contrast, the dialecticians go up from hypotheses to the Form of the Good, and see Forms as “connected in a coherent system dependent on the Good.”<sup>112</sup>

Cross and Woosley’s view is an attempt to meet the two, apparently inconsistent,<sup>113</sup> demands at once: that both the objects of *dianoia* and *noêsis* are Forms, and that the objects of *dianoia* are images of the objects of *noêsis*. On this difficult point, they argue as follows:

It is not implausible to say that seen in their isolation and fragmentariness the Forms are *different objects* from what they are when seen in their connectedness and their dependence on the Form of the Good.<sup>114</sup>

So when seen in their connectedness and their dependence on the Form of the Good, Forms show themselves as they are, but when seen in their isolation and fragmentariness, Forms show only incomplete images of what they are. This view presupposes that Forms are connected in a coherent system under the Form of the Good. If that is how the Forms exist, they cannot exist otherwise since they cannot change (479a1-3, e7-8, 484b4, 485b2-3). Now we may concede to Cross and Woosley that images of Forms

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<sup>108</sup> Except the Form of the Good, which cannot be an object of *dianoia*.

<sup>109</sup> Cross & Woosley, *Plato’s Republic*, 232.

<sup>110</sup> Cf. Cross & Woosley, *Plato’s Republic*, 244.

<sup>111</sup> Cross & Woosley, *Plato’s Republic*, 238.

<sup>112</sup> Cross & Woosley, *Plato’s Republic*, 238. Cf. Plato’s remarks, “the man who is capable of an overview is dialectical while the other who isn’t, is not” (537c7).

<sup>113</sup> For images of Forms must be less than Forms.

<sup>114</sup> Cross & Woosley, *Plato’s Republic*, 238. Emphasis added.

seen in their isolation and fragmentariness are different from the Forms seen as they are. But what I find problematic with their interpretation is their claim that those images of Forms are “objects”. The Forms are objects in the sense that they exist as they are, whether anybody sees them or not. But do those images of Forms exist when nobody sees them? They do not.<sup>115</sup> So those images of Forms are not objects.<sup>116</sup>

To summarize Cross and Woozley’s interpretation, they begin with the assumption that the only intelligible objects are Forms, and at the same time they wish to keep true to Plato’s claim that the objects of *dianoia* are images of the objects of *noêsis*.<sup>117</sup> To do that, they note the fact that the mathematicians and the dialecticians use two different methods to recognize Forms, and think that the mathematical and dialectical methods produce two kinds of cognition, incomplete and complete. Then apparently applying Plato’s principle of individuation for faculties (477c9-d5),<sup>118</sup> Cross and Woozley infer that corresponding to the two kinds of cognition, there are two kinds of objects, images and Forms. But this is a wrong application of the principle. According to Cross and Woozley, the mathematicians’ method, which produces an incomplete cognition, is itself incomplete as a method for recognizing Forms.<sup>119</sup> When an incomplete method is applied to an object, it does not follow that the object is incomplete. For example, when you make a hasty judgment about a traffic signal, you are responsible for it, and it does not follow that the signal is incomplete, nor that there is an incomplete signal besides the real signal. Although I cannot enter the detailed discussion of the principle of individuation for faculties here, the principle seems to presuppose the appropriate match between the objects and faculties. Only if the appro-

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<sup>115</sup> If they ever exist, they would do so perhaps only in the mind of somebody who sees them.

<sup>116</sup> I do not mean that no images are objects, for most images, e.g., shadows and appearances on smooth surfaces, are objects, which exist even if nobody sees them.

<sup>117</sup> Cross & Woozley, *Plato’s Republic*, 237-8.

<sup>118</sup> Cf. note 76 above, too.

<sup>119</sup> This is implied by Cross & Woozley, *Plato’s Republic*, 243-4.

priate faculties or methods are applied to objects, the difference of the cognitive states produced will indicate the difference of the objects.

For the above reasons, Cross and Woosley fail to establish that incomplete images of Forms are the objects of *dianoia*. They would have to say either that incomplete images of Forms are not objects, as I have argued above, or if they insist that those images are objects independent from our cognition and different from Forms, that there are two kinds of intelligible objects, Forms and their images. The latter option would make their view considerably similar to the traditional interpretation.<sup>120</sup>

Even though the modern interpretation is not adequate, the traditional interpretation might be worse for its own difficulties. So lastly, I shall clear the difficulties felt about the traditional interpretation. There are three passages that prompted the modern interpretation. The first one is 510d5-8, where it is said that the mathematicians think about “the square itself and the diagonal itself.”<sup>121</sup> This is considered as one of the regular expressions Plato uses to refer to Forms.<sup>122</sup> If so, the expression would suggest that the objects of *dianoia* are Forms. But this type of expression does not necessarily refer to Forms.<sup>123</sup> It is simply an expression to indicate that what is referred to is not an image but an original, and exactly what the original is can vary according to the context. For example, “Socrates himself” can refer to the actual sensible Socrates, not the Form of Socrates, insofar as the actual Socrates is conceived as the original of his images. In the present case, the square the mathematicians think about is called “the square itself” in distinction from its sensible images, and the square itself can be a mathematical square.

The second passage is 511d1-2, where it is said that “the mathematicians do not

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<sup>120</sup> Though the question of exactly what the images of Forms are remains.

<sup>121</sup> Similar expressions occur at 524e6 (“the one itself”) and 525d6 (“numbers themselves”).

<sup>122</sup> Cf. Cross & Woosley, *Plato’s Republic*, 236.

<sup>123</sup> Cf. Burnyeat, “Platonism and Mathematics,” 219-20, note 19.



possess *noêsis* with respect to their objects even though these can be objects of *noêsis*, given a beginning.” This suggests that the objects of mathematics are the same as those of *noêsis*, and that the same Forms can be objects of *dianoia* when recognized by the mathematicians, but can be objects of *noêsis* when recognized by the dialecticians.<sup>124</sup> The suggestion presupposes that if the objects of mathematics were intermediates, they cannot be objects of *noêsis*. Certainly Plato says, for example, that as knowledge and opinion are different faculties, their objects are different (477e8-478a5), and that the objects of opinion cannot be known (507b9). But the matter is not so simple. There is quite an ambiguity about the objects of the mathematicians’ cognition; for their cognitive state is *dianoia*, and they “dream about what is” (533b8-c1), i.e., “believe a likeness of something to be not a likeness, but rather the thing itself to which it is like” (476c6-7).<sup>125</sup> The ambiguity is hard to disentangle,<sup>126</sup> but the above quoted sentence at issue is sufficiently natural, and I want to make its meaning clear. First let us look at the people who see an image of Socrates and whose cognitive state is *eikasia*. From their point of view, they believe that they see Socrates himself. But from the point of view of *pistis*, they fail to see the object they believe to see, even though the object can be seen. In the same way, the sentence at issue can be glossed as follows. From their point of view, the mathematicians believe that they know the square itself and the diagonal itself (510d7-8). But from the point of view of *noêsis*, they fail to know the objects they believe to know, even though the objects can be known by *noêsis*. Understood this way, the sentence does not mean that the mathematicians do not possess *noêsis* with respect to the mathematical intermediates, nor implies that the mathematical intermediates can be objects of *noêsis*.

The third passage is 534a5-8, which reads:

<sup>124</sup> Cf. Ross, *Plato’s Theory*, 60.

<sup>125</sup> Needless to say, within the intelligible realm.

<sup>126</sup> For the difficulties involved in the concept of mistaking something for something else, see the discussion on the possibility of false judgments in *Theaetetus* 187a-200c.

But as for the proportion between the things over which these [*noêsis*, *dianoia*, *pistis*, and *eikasia*] are set and the division into two parts of each—the opinable and the intelligible—let’s let that go, Glaucon, so as not to run afoul of argument many times longer than those that have been gone through.

This passage is sometimes interpreted as meaning that “Plato did not know what to say about them [the two sorts of intelligible object],” because there is no proportion of them, i.e., they are not ontologically different.<sup>127</sup> Certainly Plato there says that he will “let go” the proportion and division of objects. But “let go” does not mean to give up or abandon or anything like that. It simply means in the passage to let go the proportion and division of objects *undiscussed*. And in the passage Plato explicitly refers to, and recognizes, what he proposes to let go undiscussed, i.e., the proportion and division of objects. So the passage rather supports the traditional interpretation. Further Plato gives the reason for not discussing the proportion and division of objects: it will involve “argument many times longer than those that have been gone through.” This is a good reason, since the *Republic* is a dialogue primarily concerned with justice, and the digression into metaphysical issues should be kept minimum.<sup>128</sup> Actually the objects of *eikasia* and *pistis* were briefly discussed in 509e1-510b1, so the difficult part that requires “argument many times longer”, will be the objects of *dianoia* and *noêsis*, and the proportions that involve them.<sup>129</sup> These are very difficult issues to be avoided in a political dialogue.

Once those difficulties felt about the traditional interpretation have been cleared, there is no reason to shy away from it.

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<sup>127</sup> Robinson, *Plato’s Earlier Dialectic*, 193-5. Ross, too, thinks that the distinction between the objects of *dianoia* and *noêsis* is not “considered important enough to be worth repeating” (*Plato’s Theory*, 69).

<sup>128</sup> Cf. Brentlinger, “The Divided Line,” 161-2.

<sup>129</sup> Including the proportion of the objects of *noêsis* and *pistis* and that of the objects of *dianoia* and *eikasia*.

## 6. Conclusion

In this paper, I have discussed the four major interpretative problems of the Simile of the Line. In my discussions of them, I have argued: (1) the visible represents the sensible in general and the opinable; (2) *eikasia* means taking an image for its original; (3) the equality of DC and CE is an unintended consequence of the mechanism of the Simile; and (4) the objects of *dianoia* are mathematical intermediates. The overall picture of the Simile my discussion brings out is that the purpose of the Simile is first to illustrate the connection between the visible and the intelligible by the relation between images and their originals, and second to distinguish mathematical intermediates from Forms within the intelligible.

The problem that is left out of the present paper is the mathematician's hypothetical method and the philosopher's dialectical method. That difficult issue has to wait for another paper.

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